D-brane Instantons and Flavour Physics

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Abstract

In this master thesis D-brane-instanton generated contributions to the phenomenology of intersecting D-brane models are investigated. At first a brief introduction to Calabi-Yau and orientifold compactification of supersymmetric string theory is given and the elementary properties and consistency conditions of intersecting brane models like Ramond-Ramond-tadpole cancellation and Green-Schwarz mechanism are explained. The connection between field theory- and D-instantons is shown by the example of the D3/D(-1)-system and the instanton calculus in string theory is described which is the main tool to calculate instanton-contributions to the effective supergravity action. The normalization of the holomorphic contributions in terms of Planck- and string-mass is derived. After showing the need for D-brane instantons in the generation of perturbatively forbidden Yukawa couplings in MSSM orientifold models, the implications of the required "Yukawa-instantons" for flavour violating effects are investigated systematically for a specific 5-stack quiver model. Contributions to lepton-flavour violating meson decays are found which are finally used to derive a lower bound on the string mass scale.

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Chapter 1

Introduction

The standard model of particle physics is one of the great achievements of theoretical physics. As a renormalizable, anomaly free quantum gauge field theory containing QCD and electroweak theory it can explain with extraordinary accuracy a wide variety of phenomena ranging over many orders of magnitude in energy. It explains the dynamics and systematics of the strong force responsible for mesons and baryons, the vector-axial-nature of the weak force and predicted the properties of the massive W and Z gauge bosons that mediate charged and neutral currents. It explained the supression of flavour changing neutral currents leading to the prediction of the charm quark. And it also gives a candidate mechanism for electroweak symmetry breaking -just to mention a few of the deep insights of this model of nature.

But it is also clear that it cannot be a complete theory of nature. First of all it does not include gravity. But also in its attempt to describe the other three fundamental interactions, there are a lot of facts which cannot be explained by the standard model. There are at least 26 undetermined parameters, where no predictions can be made from the theory. For example the masses of the elementary fermions are set by the Yukawa couplings but we do not know what determines the values of these couplings. There is no prediction from the standard model. Experimentally, we can determine the masses of quarks and leptons. They range over many orders of magnitude and there are huge differences between the families. The standard model also does not explain why there are three families with different mass hierarchies and also anomaly cancellation seems a big coincidence.

String theory is not only a promising candidate to get a unified quantum theory of gravity and the other interactions but can give a *geometrical* explanation of some of the above mentioned problems. The consideration of the quantum theory of a one-dimensional object propagating in spacetime (which is parameterized by a map of a two dimensional domain of definition to a target manifold) makes it possible to have the right amount of degrees of freedom to have both, gravitational and gauge excitations in the massless spectrum. To be a consistent quantum theory, (supersymmetric) string theory predicts spacetime to be ten-dimensional and therefore we have to explain why we have not discovered the extra dimensions up to now. One way to do this is to assume spacetime to be the direct product of the visible Minkowski space and an internal compact (Calabi-Yau) manifold. The geometry of the internal space will then determine some of the physical properties of the external space.

One important discovery in string theory was the existence of non-perturbative objects like D-branes. It was shown that that the massless spectrum (which is associated to the particles we see in nature) of open strings ending on a number of coincident branes reproduces the degrees of freedom of non-Abelian super Yang-Mills theories. This is the basis of intersecting brane models in which branes contain the external Minkowski space and intersect in the compact internal space. Depending on the intersection geometry we get different realizations of four dimensional chiral fermions. Multiple intersections of two stacks of branes correspond to the families of particles and from general consistency conditions like charge conservation in the compact space one can infer anomaly cancellation.

Furthermore it is possible to relate the strength of the Yukawa couplings to the volume of internal geometrical objects such as world-sheet- and D-instantons which opens up the possibility to explain the observed mass hierarchies of the standard model. Conversely these hierarchies together with phenomenological bounds like proton or meson decay give us important restrictions and hints about the size and energy scales of the fundamental parameters in string theory.

Chapter 2

Orientifold compactifications and D-branes

2.1 Calabi-Yau compactification

The cancellation of the superconformal anomaly requires the spacetime dimension of superstring-theories to be $D = 10$. Since we observe only 4 dimensions in nature (at least up to the available energies in present accelerators) we have to "make the remaining dimensions small". One way to achieve this is to demand our spacetime to be of a direct product form $M_{10} = M_4 \times \mathcal{X}$ where M_4 should be four dimensional Minkowski space and the internal space $\mathcal{X}$ should be a compact six-dimensional manifold, about which we make reasonable physical and geometrical assumptions to be described in the following subsection.

2.1.1 Geometrical and physical requirements

One way to deal with the hierarchy between the electroweak and GUT/Planck scales is to avoid quadratic divergences in certain loop corrections to the standard model. This can be done by demanding the standard model to be supersymmetric because then fermionic and bosonic contributions cancel out. Therefore we have to choose our internal manifold such that spacetime supersymmetry in four dimensions is possible. We start with a direct product metric of the form

$$ds^2 = g_{\mu\nu} dx^\mu \otimes dx^\nu + g_{mn} dy^m \otimes dy^n \tag{2.1.1}$$

Here, the internal coordinates on $\mathcal{X}$ are labeled by y^m and the external coordinates on M_4 by x^μ and capital coordinate indices range over the whole product manifold $M = 0, 1 \ldots 9$. One could also include a "warp factor" multiplying the first or the second summand of (2.1.1) but we restrict to the above case. We consider the low-energy limits of type IIA/B superstring theory which are given by type IIA/B supergravity in 10 dimensions. To have unbroken supersymmetry, the vacuum should obey $\epsilon Q|0\rangle = 0$, where Q is the generator of the ten-dimensional supersymmetry-transformation parameterised by ϵ and therefore both objects are spinors of $SO(1,9)$. If we denote by Ψ an arbitrary field we get the following condition on the vacuum expectation values

$$\langle \delta_\epsilon \Psi \rangle = \langle 0|[\epsilon Q, \Psi]|0\rangle = 0 \tag{2.1.2}$$

Now consider the variations of the gravitinos $\psi_M = \binom{\psi^1_M}{\psi^2_M}$ in type II supergravity :

$$\delta_\epsilon \psi_M = \nabla_M \epsilon + \frac{1}{4} H_{MNP} \Gamma^{NP} \mathcal{P} \epsilon + \dots \tag{2.1.3}$$

Here the Γ-matrices represent the Clifford algebra in ten dimensions, $H = dB$, where B is the NS-NS antisymmetric tensorfield and $\mathcal{P}$ is Γ_{11} for type IIA and σ_3 for type IIB (omitting the factors of identity). The dots represent terms involving the different R-R-fields. If we take the vacuum expectation value of all the bosonic fields to be zero (and thus $H = 0$), we get to the constraint (the covariant derivative contains the vacuum expectation value of the spin connection):

$$\delta_\epsilon \psi_M = \nabla_M \epsilon = 0 \tag{2.1.4}$$

Since the spinor-bundle over the product manifold $M_4 \times \mathcal{X}$ decomposes into tensor products of the corresponding bundles over M_4 and $\mathcal{X}$ respectively, we get the following decompositions in type IIA/B

$$\epsilon^1_{IIA} = \xi^1_+(x^\mu) \otimes \eta_+(y^m) + \xi^1_-(x^\mu) \otimes \eta_-(y^m) \tag{2.1.5}$$

$$\epsilon^2_{IIA} = \xi^2_+(x^\mu) \otimes \eta_-(y^m) + \xi^2_-(x^\mu) \otimes \eta_+(y^m) \tag{2.1.6}$$

$$\epsilon^a_{IIB} = \xi^a_+(x^\mu) \otimes \eta_+(y^m) + \xi^a_-(x^\mu) \otimes \eta_-(y^m) \qquad a = 1,2 \tag{2.1.7}$$

Now we can expand ∇_M into internal and external part and infer from equations 2.1.5 to 2.1.7

$$\partial_\mu \xi^{1/2}_\pm(x) = 0 \qquad \nabla_m \eta_\pm(y) = 0 \tag{2.1.8}$$

Therefore we get a constant spinor in four dimensions and we can choose for example η_+ as a covariantly constant Weyl spinor on our internal manifold $\mathcal{X}$. This has interesting consequences to the type of the internal manifold one can choose:

- $\mathcal{X}$ is a Kähler manifold. For the proof we observe that we can define the following endomorphism of the tangent bundle of $\mathcal{X}$:
$$J_m^n = i\eta_+^\dagger \Gamma_m^n \eta_+$$
where $\Gamma_m^n = \frac{1}{2}(\Gamma_m\Gamma^n - \Gamma^n\Gamma_m)$, and the Γ_n are this time the 6-dimensional gamma-matrices. From their properties, one can show that J squares to $-\text{Id}$ and the metric on $\mathcal{X}$ is Hermitean w.r.t. J (for more details see for example [2]). In addition, we see immediately that the Nijenhuis tensor of J vanishes (because η_+ is covariantly constant):
$$N_{mn}^p = J_m^q(\nabla_q J_n^p - \nabla_n J_q^p) - J_n^q(\nabla_q J_m^p - \nabla_m J_q^p) = 0$$
And thus J is integrable.
- $\mathcal{X}$ is Ricci-flat. For the proof we use again that η_+ is covariantly constant:
$$0 = [\nabla_m, \nabla_n]\eta_+ = \frac{1}{4}R_{mnpq}\gamma^{pq}\eta_+$$
from which it follows for Riemannian manifolds that $R_{mn} = 0$.

From these two facts it follows from a theorem by Iwamoto that the holonomy group of $\mathcal{X}$ is contained in $SU(3)$ (see [2]). Because η_+ is a spinor on the internal manifold, its chirality components transform as $\mathbf{4}$ and $\bar{\mathbf{4}}$ under $SO(6) \simeq SU(4)$ if we parallel transport it around loops in $\mathcal{X}$. Thus if we assume the holonomy is $SU(3)$ i.e. we have a Calabi-Yau-manifold we can decompose

$$\mathbf{4}_{SU(4)} = \mathbf{3}_{SU(3)} \oplus \mathbf{1}_{SU(3)} \tag{2.1.9}$$

and so we get exactly one singlet, i.e. a constant spinor on $\mathcal{X}$ and the same for the $\bar{\mathbf{4}}_{SU(4)}$. If we again use the decomposition (2.1.5) to (2.1.7) we see that we get in this case one constant Majorana spinor in four dimensions, which parameterizes $\mathcal{N} = 1$ supersymmetry in four dimensions. If we instead considered a proper subgroup of $SU(3)$ there would in general be more singlets and therefore extended supersymmetry in four dimensions. For example if we consider type II theories, we start with $\mathcal{N} = 2$ in $D = 10$ and depending on the internal geometry we get in four dimensions $\mathcal{N} = 8$ if $\mathcal{X}$ is a torus T^6, $\mathcal{N} = 4$ if $\mathcal{X} = K3 \times T^2$ and as already mentioned $\mathcal{N} = 2$ for $\mathcal{X} = CY_3$.

2.1.2 Compactification of type II theories on Calabi-Yau manifolds

As shown in the last subsection, we can get the minimal amount of supersymmetry in four dimensions if we compactify on Calabi-Yau threefolds $\mathcal{X}$. We now want to summarize elementary facts about Calabi-Yau manifolds and then give the massless spectrum of string theory compactified on this type of geometry.

Calabi-Yau manifolds

For the basics of the geometry of Calabi-Yau manifolds we refer to the standard literature, for example [3] to [5].
There are five equivalent definitions of a Calabi-Yau manifold. A Kähler manifold of complex dimension n

- which is Ricci-flat
- which has vanishing first Chern class
- which has trivial canonical bundle
- with holonomy group $SU(n)$
- which has a globally defined, non-vanishing holomorphic n-form.

is called Calabi-Yau.

We are now going to list some elementary properties of this class of manifold which will be important later.

If we denote the space of sections of $\bigwedge^{p,q}(\mathcal{X}) := \bigwedge^p T^{*(1,0)}\mathcal{X} \otimes \bigwedge^q T^{*(0,1)}\mathcal{X}$ by $\Omega^{p,q}(\mathcal{X})$ then the Dolbeault cohomology is defined by

$$H^{p,q}_{\bar{\partial}}(\mathcal{X}) := \frac{\mathrm{Ker}(\bar{\partial} : \Omega^{p,q}(\mathcal{X}) \to \Omega^{p,q+1}(\mathcal{X}))}{\mathrm{Im}(\bar{\partial} : \Omega^{p,q-1}(\mathcal{X}) \to \Omega^{p,q}(\mathcal{X}))}. \tag{2.1.10}$$

We then can define the Hodge numbers $h^{p,q} := \mathrm{dim} H^{p,q}(\mathcal{X})$. It turns out ([3]) that the general Hodge diamant of a Calabi-Yau threefold is given by

$$\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & h^{1,1} & & 0 & \\
1 & & h^{2,1} & & h^{2,1} & & 1 \\
 & 0 & & h^{1,1} & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array} \tag{2.1.11}$$

The two nontrivial Hodge-numbers $h^{1,1}$ and $h^{2,1}$ have an important geometrical interpretation in terms of the moduli of the Calabi-Yau manifold. To see this, we add to the metric a perturbation $g_{\mu\nu} + h_{\mu\nu}$ and demand that the Calabi-Yau-condition is preserved (in order to get perturbations of the metric which are not related by diffeomorphism, we also demand to fix diffeomorphism-invariance $\nabla^\mu h_{\mu\nu} = 0$).

- $\mathbf{h^{1,1}}$
  Consider the Kähler form ω of $\mathcal{X}$: $\omega = g_{\mu\bar{\nu}} dz^{\mu} \wedge d\bar{z}^{\bar{\nu}}$. Looking at perturbations of the form $h_{\mu\bar{\nu}}$ one can infer from the Lichnerowicz-equation [10]

  $$\nabla^{\lambda}\nabla_{\lambda} h_{\mu\nu} + 2R_{\mu\ \nu}^{\ \alpha\ \beta} h_{\alpha\beta} = 0 \qquad (2.1.12)$$

  that ω changes by a harmonic form, and therefore $\omega + \delta\omega$ again gives rise to a Kähler class $[\omega + \delta\omega]$. Now Yau's theorem ([6], [7]) states, that, provided vanishing first Chern class, there is a unique element in each Kähler class of $\mathcal{X}$ that is Ricci-flat. Thus the cohomology $H^{1,1}(\mathcal{X})$ is isomorphic to the possible varitations of the Kähler form that preserves the Calabi-Yau condition and therefore the moduli space of Kähler structures has dimension $h^{1,1}$.

- $\mathbf{h^{2,1}}$
  In this case we look at perturbations of the form $h_{\mu\nu}$. These transformations correspond to a change of the complex structure J on the manifold $\mathcal{X}$ (for more details see for example [8]). If we change the coordinates on $\mathcal{X}$ infinitesimally from z to z', generated by a vector field $V = v^{z_i}\partial_i + \bar{v}^{\bar{z}_i}\bar{\partial}_i$, J transforms as

  $$J \longrightarrow AJA^{-1} \quad A = \mathbf{1} + \mathcal{J} \qquad (2.1.13)$$

  where $\mathcal{J}$ is the Jacobian of the transformation. Therefore, infinitesimally (this is just a local statement, so actually we get information only about the tangent space to the moduli space of complex structures at the point J)

  $$J' = J + \partial\bar{v} + \bar{\partial}v \qquad (2.1.14)$$

  So these perturbations are given by elements of $H^1_{\bar{\partial}}(T\mathcal{X})$ for the holomorphic part and by $H^1_{\partial}(T\mathcal{X})$ for the antiholomorphic part which are given by $H^1(T\mathcal{X})$ by the Cech-Dolbeault-isomorphism ([8]). Taking into account the triviality of the canonical bundle $\wedge^3 T^*\mathcal{X}$ and therefore the triviality of $\wedge^3 T\mathcal{X} \cong \wedge^3 T^*\mathcal{X}$ we see that the wedge product gives an isomorphism between $\wedge^2 T^*\mathcal{X}$ and $\wedge T^*\mathcal{X}$ and therefore

  $$H^1(T\mathcal{X}) \cong H^1(\wedge^2 T^*\mathcal{X}) \cong H^{2,1}(\mathcal{X}) \qquad (2.1.15)$$

  Thus we see that at least locally, the moduli space of complex structures of $\mathcal{X}$ is characterized by $H^{2,1}(\mathcal{X})$ and has therefore dimension $h^{2,1}$.

We remark that a fascinating aspect about Calabi-Yau manifolds is, that lots of them come in mirror-pairs: $(\mathcal{X}, \mathcal{X}')$ is said to be a mirror pair, if $H^{2,1}(\mathcal{X}) \cong H^{1,1}(\mathcal{X}')$ and $H^{2,1}(\mathcal{X}') \cong H^{1,1}(\mathcal{X})$. The mirror conjecture says that this holds for all Calabi-Yau manifolds. More on this interesting subject and also on the relation to physics can be found in [9] and [10].

Compactification of type II strings on Calabi-Yau manifolds

For later reference we give the massless spectra of the type II theories after GSO projection (which in the NS sector eliminates the tachyon and leaves the massless states unchanged and in the R sector selects a specific chirality of the ground state). Let us denote the creation and annihilation operators by ψ_n (half integer for NS and integer for the R sector). To disinguish left and right movers, we use a tilde. We begin with the IIA theory and comment about the changes in the IIB case:

- *NS-NS*:
  $$\psi^i_{-\frac{1}{2}}\tilde{\psi}^j_{-\frac{1}{2}}|0,k,NS\rangle_L \otimes |0,k,NS\rangle_R$$
  which can be decomposed as
  $$\mathbf{8}_V \otimes \mathbf{8}_V \cong \mathbf{35} \text{ (graviton) } \oplus \ \mathbf{28} \text{ (B-field) } \oplus \ \mathbf{1} \text{ (dilaton)}$$
  which is the same for the IIB theory.
- *R-NS*
  $$|0,k,\vec{\alpha},R\rangle_L \otimes \tilde{\psi}^i_{-\frac{1}{2}}|0,k,NS\rangle_R$$
  which include the states
  $$\mathbf{8}_S \otimes \mathbf{8}_V \cong \mathbf{56} \text{ (gravitino, pos.chirality) } \oplus \ \mathbf{8} \text{ (dilatino, neg.chirality)}$$
  which is again the same for the IIB theory.
- *NS-R*
  $$\psi^i_{-\frac{1}{2}}|0,k,NS\rangle_L \otimes |0,k,\vec{\alpha},R\rangle_R$$
  which gives the states
  $$\mathbf{8}_V \otimes \mathbf{8}_S \cong \mathbf{56} \text{ (gravitino, neg.chirality) } \oplus \ \mathbf{8} \text{ (dilatino, pos.chirality)}$$
  Here, we have the opposite chiralities for the IIB theory.
- *R-R*
  $$|0,k,\vec{\alpha},R\rangle_L \otimes |0,k,\vec{\beta},R\rangle_R$$
  which contains the the following states
  $$\mathbf{8}_S \otimes \mathbf{8}_S \cong \mathbf{8} \text{ (1-form) } \oplus \ \mathbf{56} \text{ (3-form)}$$
  In contrast to the odd dimensional forms we get in the IIB theory the even dimensional forms
  $$\mathbf{1} \text{ (0-form) } \oplus \ \mathbf{28} \text{ (2-form) } \oplus \ \mathbf{35} \text{ (4-form)}$$

The massless fields are the fields of the type IIA/B supergravities. In the following we want to compactify these theories to four dimensions, with a Calabi-Yau threefold as internal geometry. To get the four dimensional low energy effective theory, we have to dimensionally reduce, like in [1] [12] [11] the fields in the above list. For definiteness, consider the type IIA theory (similar arguments hold for the IIB case). In general, if we have the massless Dirac/Klein-Gordon equation in ten dimensions, we can split them into four dimensional and internal part, the internal part describing the four dimensional mass.

- For ten-dimensional scalar fields, the internal operator is the Laplace-operator. The Laplace-equation on the compact internal space has just the constant solution and therefore we get one scalar field in four dimensions.
- For the p-form fields C_p in the ten-dimensional spectrum which have an action

  $$S_p = \int_{\mathcal{X}} dC_p \wedge *dC_p \tag{2.1.16}$$

  and for the equations of motion, we get $\triangle C_p = 0$, where $\triangle = d * d + dd*$ is the internal Laplace-Beltrami operator (we fixed the gauge by imposing $d * C_p = 0$). Thus the number of massless fields in four dimensions is given by the number of harmonic forms or according to Hodge's theorem by the dimension of the corresponding cohomology group.
- If we have spinor fields in ten dimensions we can make the ansatz (where the ξ_n are eigenfunctions of the internal Dirac operator)

  $$\Psi(x^\mu, y^m) = \sum_n \zeta_n(x^\mu) \otimes \xi_n(y^m) \tag{2.1.17}$$

  and decompose the Dirac operator into internal and external part. Again the zero modes of the internal operator determine the massless fields in four dimensions. In this case the kernel of the internal operator has dimension given by the index of the internal Dirac operator.
- Finally the ten-dimensional metric G_{MN} splits into various sectors. If we take both indices four dimensional, $g_{\mu\nu}$, which is a scalar on the internal space, we get just the four dimensional metric. The index structures $g_{\mu j}$ and $g_{\mu\bar{j}}$ would correspond to $(1,0)$- and $(0,1)$-forms on the internal space, respectively. But from the Hodge-diamond in the previous section, we can see, that the dimensions of the corresponding cohomologies are zero. The remaining metric components with all indices internal, g_{ij} and $g_{i\bar{j}}$ are scalars in four dimensions. The four dimensional masses correspond to the variations of the internal metric, and as we saw in the last section, the massless cases are exactly the complex structure ($h^{2,1}$ complex scalar fields) and Kähler moduli ($h^{1,1}$ real scalar fields). One can physically interpret them as the Goldstone bosons coming from the choice of one specific vacuum geometry [1].

We can now apply this procedure to the type IIA supergravity theory. To see the structure of the four dimensional theory, we can organize the resulting fields into representations of four dimensional $\mathcal{N} = 2$ supersymmetry. To find the spinor-part of the four dimensional multiplets, we use the following decomposition of the spin-bundles $S(M_4 \times \mathcal{X}) = S^+(M_4 \times \mathcal{X}) \oplus S^-(M_4 \times \mathcal{X})$in ten dimensions:

$$\begin{aligned} S(M_4 \times \mathcal{X}) =& (S^+(M_4) \otimes S^+(\mathcal{X})) \oplus (S^-(M_4) \otimes S^-(\mathcal{X})) \\ & \oplus (S^-(M_4) \otimes S^-(\mathcal{X})) \oplus (S^+(M_4) \otimes S^+(\mathcal{X})) \end{aligned} \tag{2.1.18}$$

From this we get the decomposition of spin-$\frac{1}{2}$-matter. For the spin-$\frac{3}{2}$-matter we have a similar decomposition into different parts (see for example [12]). Instead of writing down the precise formulas (which we do not need in the following) we want to describe the physical interpretation.

- The four-dimensional part of the metric and one-form together with the two chiralities of the two spin-$\frac{3}{2}$ parts of the decomposition of the gravitini give the four dimensional gravity-multiplet.

- The $h^{1,1}$ zero modes of the metric get complexified by the $(1,1)$ part of the NS-NS two-form and together with the $C_{\mu i \bar{j}}$ part of the R-R three-form and four dimensional spin-$\frac{1}{2}$ parts of the gravitini give $h^{1,1}$ vector-multiplets.

- The $h^{2,1}$ complex zero modes of the metric together with the $C_{ij\bar{k}}$ and $C_{\bar{i}\bar{j}k}$ part of the R-R three-form and two chiralities of the spin-$\frac{1}{2}$ part of the decomposition of the gravitini give $h^{2,1}$ complex hypermultiplets.

- The zero mode of the dilaton combines with the dual of the four-dimensional part of the NS-NS two-form (which gives rise to a scalar c given by $*_{4d}dB = dc$). Together with the four parts of the decomposition of the dilatini and the C_{ijk} and $C_{\bar{i}\bar{j}\bar{k}}$ parts of the R-R three-form to an additional hypermultiplet which is also called the universal hypermultiplet, because it is also part of the spectrum of the type IIB theory.

We observe that there are $h^{1,1}$ vectormultiplets and $h^{2,1} + 1$ hypermultiplets and the gravity multiplet in type IIA. The massless four dimensional spectrum of the type IIB theory can be obtained with similar methods (see [12]). An interesting fact is that the spectra of type IIA/B theories differ only by exchanging the Hodge numbers $h^{2,1}$ and $h^{1,1}$. This is again a consequence of mirror symmetry, which states in general that compactification of type IIA on a Calabi-Yau threefold gives the same spectrum as compactification of type IIB on the mirror threefold.

2.2 D-branes

One of the main sources of the second superstring revolution was the observation of non-perturbative objects, especially Dirichlet-branes. They are not present in the perturbative string spectrum but show up in the easiest case if we consider the T-dual of string theory compactified on a circle. The observation is that open string endpoints in the T-dual theory are confined (have Dirichlet boundary conditions) to p-dimensional submanifolds of the target space. But this is not only a geometric statement. In fact, Polchinski showed [15] that one can describe the fluctuations around the submanifold in terms of quantized open strings ending on it and therefore a D-brane is a "dynamical" object. The resulting open string spectrum contains a $U(1)$-gauge connection and it turns out that if N Dp-branes coincide, we get a $U(N)$ gauge theory. Therefore D-branes and as we will see later, intersecting branes (where one also gets chiral matter fields) are a natural starting point for constructing models of particle physics.

2.2.1 T-duality of open strings

Consider for simplicity bosonic string theory with target space topology $\mathbf{R}_{1,24} \times S^1$, the radius of the circle being R. We get a discrete momentum along the compact direction. In the *closed* string sector we have in addition a conserved winding number k and the mass spectrum is given by

$$M^2 = \frac{n^2}{R^2} + \frac{k^2 R^2}{\alpha'^2} + \frac{2}{\alpha'}(N + \tilde{N} - 2). \tag{2.2.19}$$

we see that this spectrum is invariant under the T-duality transformation

$$R \longrightarrow \frac{\alpha'}{R} \tag{2.2.20}$$

which interchanges winding and momentum quantum numbers. Now in the *open* string, there is no conserved winding since differnt winding states can be deformed continuously into each other. And therefore the question arises, what is the T-dual of the open strings? Remembering that T-duality has the following effect ([11]) on the embedding coordinate $X^{25}(\tau, \sigma)$:

$$X^{25}(\tau,\sigma) = X_L(\tau+\sigma) + X_R(\tau-\sigma) \longrightarrow \tilde{X}^{25}(\tau,\sigma) = X_L(\tau+\sigma) - X_R(\tau-\sigma) \tag{2.2.21}$$

we observe that Neumann and Dirichlet boundary conditions get exchanged:

$$\partial_\sigma X^{25} \longleftrightarrow \partial_\tau \tilde{X}^{25} \tag{2.2.22}$$

Thus an open string with Neumann conditions on the compact coordinate will be confined to a 25-dimensional submanifold in the dual theory. It is also interesting to consider the limits of large and small radius. Taking $R \to 0$ in the *closed* sector, we see from 2.2.19 that all nonzero momentum states will decouple but we get a continuous set of winding modes. Therefore we actually get again a theory in 26 dimensions (we interprete the continuous set of winding modes as new free momentum states). The other limit $R \to \infty$ has a similar effect and the two scenarios are physically mapped to each other by T-duality. In the *open* sector $R \to 0$ will also decouple states with nonzero momentum in the compact direction but there will be no continuum coming from winding states. Therefore we only get a theory in 25 dimensions. The only difference of open and closed strings are the boundaries and therefore the interpretation is that the open string boundaries get confined to a lower dimensional submanifold, a D-brane (see figure 2.1).

2.2.2 Gauge symmetry

On $\mathbf{R}_{1,24} \times S^1$ we can decompose the gauge field (of the open bosonic string massless spectrum) as (A^μ, A^{25}). By a gauge transformation we can chose A^{25} to be a constant denoted by $\frac{\theta}{2\pi R}$. The momentum in the 25-direction gets shifted to

$$p^{25} + A^{25} = \frac{n}{R} + \frac{\theta}{2\pi R} \tag{2.2.23}$$

Taking in addition $U(n)$-Chan Paton lables $|ij\rangle$ into account, then we get again a shifted momentum and therefore the changed mass formula:

$$M^2 = \frac{1}{2\pi R}(2\pi n + \theta_i - \theta_j) + \frac{1}{\alpha'}(N - 1) \tag{2.2.24}$$

In the generic case ($\theta_i \neq \theta_j$ for $i \neq j$) we get a $U(1)^n$ gauge theory. Let us then look at the T-dual theory. Using the mode expansion ([11]) we get (using $\tilde{X}^{25} = \tilde{X}^{25} + 2\pi\tilde{R}$)

$$\tilde{X}^{25}(\pi) - \tilde{X}^{25}(0) = -(2\pi n + \theta_i - \theta_j)\tilde{R} = -2\pi\alpha'(A^{25}_{ii} - A^{25}_{jj}) \tag{2.2.25}$$

and we see that the shift in the momentum gives an open string in the dual picture which stretches between two D-branes at positions A^{25}_{ii} and A^{25}_{jj} (see figure 2.1). Now turning the argumentation around, we see, that if we let k different D-branes coincide, we get k^2 gauge fields in the massless spectrum of the open string (because $\theta_i = \theta_j$ for the respective pairs i, j), forming the adjoint representation of $U(k)$. This is also named a *k-stack* of D-branes. The same argumentation continues to

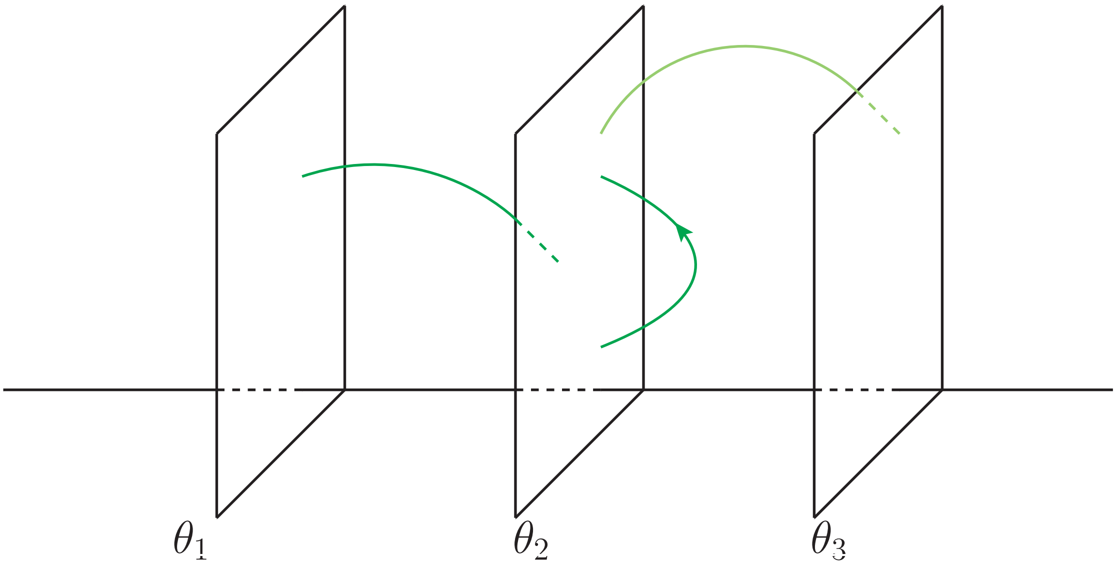


Figure 2.1: Open strings with ends on D-branes

hold if we consider superstrings and D-branes which are (one also says which wrap) more complicated submanifolds such as the product of Minkowski space and some nontrivial cycle in the internal Calabi-Yau manifold.

2.2.3 DBI action and Chern-Simons couplings

Information about the dynamics of D-branes can be derived from scattering amplitudes ([11]). The first part of the action which we will write down just for completeness is given by the Dirac-Born-Infeld-action

$$S_{DBI} = T_p \int_{\Sigma_{p+1}} d^{p+1}\xi e^{-\Phi(X)} (-\det(g_{ab}(X) + 2\pi\alpha' F_{ab}(X) - B_{ab}(X))^{\frac{1}{2}} \qquad (2.2.26)$$

where $g_{ab}(X) = \partial_a X^M \partial_b X^N g_{MN}$ and $B_{ab} = \partial_a X^M \partial_b X^N B_{MN}$ are the pullbacks of the target space metric and NS-B-field and F is the field strength of the $U(1)$-gauge field on the brane.

More important for later is the second part of the action, which describes the coupling of the brane to the Ramond-Ramond-fields. The main idea of its construction is that chiral fermions on the D-brane world volume (which for example exist at the intersections of different branes) have anomalous gauge transformations which are proportional to the index of the Dirac operator on the D-brane world volume Σ. The index can be expressed in terms of topological quantities due to the Atiyah-Singer-theorem [16]:

$$\mathrm{ind}(i\not{D}) = \int_{\Sigma} \mathrm{ch}(F) \wedge \mathrm{ch}(-F) \wedge \frac{\hat{\mathrm{A}}(T\Sigma)}{\hat{A}(N\Sigma)} \wedge \mathrm{e}(N\Sigma) \tag{2.2.27}$$

where $T\Sigma$ and $N\Sigma$ denote the tangent and normal bundles, ch the Chern character, $\hat{\mathrm{A}}$ is the $\hat{A}$-class (both explained in the first appendix), e denotes the Euler class and F is the curvature of the Chan-Paton bundle with base Σ.

Assuming now that the RR-fields C_l couple in the following way to the world volume of the D-brane (the sum over different grades of forms is denoted by $\oplus$)

$$S_{RR} = \int_{\Sigma} \bigoplus_{p} C_{p+1} \wedge Y \tag{2.2.28}$$

where Y should be of the form that the behaviour of this type of coupling under gauge transformations exactly cancels the anomaly mentioned above, then one finds the following Chern-Simons-coupling of the RR fields to the D-brane [17]:

$$S_{CS} = -\mu_p \int_{\Sigma} \mathrm{ch}(F) \wedge \sqrt{\frac{\hat{A}(R_T)}{\hat{A}(R_N)}} \wedge \bigoplus_{p} C_p \tag{2.2.29}$$

Here, R_T and R_N are the curvatures of the tangent and normal bundles to Σ and $\mu_p = 2\pi l_s^{-p-1}$, where l_s is the string length. The general action of the RR-p-form fields is given by the usual kinetic term and the above Chern-Simons action:

$$S_{C_{p+1}} = \int_{M_{1,4} \times \mathcal{X}} dC_{p+1} \wedge * dC_{p+1} \quad + S_{CS}. \tag{2.2.30}$$

we will use special cases of this action later to derive important consistency conditions for string theoretic models for particle physics.

2.3 Orientifold compactification

The massless modes of the type IIA/B superstring theories give the spectra of $\mathcal{N} = 2$ supergravity theories. In section 2.1 we observed that compactifying them on a Calabi Yau threefold gives $\mathcal{N} = 2$ supergravity theories in four dimensions. To get phenomenologically attracive models, we have to reduce the amount of supersymmetry to $\mathcal{N} = 1$ (the $\mathcal{N}$=2 chiral supermultiplet contains in addition to the standard model fermion field also a fermionic partner in the same representation of the gauge group). We also want to consider D-branes as building blocks of MSSM-like models. As we have seen in the previous section, they are charged objects under RR

antisymmetric tensor fields. Because the internal space of our compactification is compact and without boundary, we need to cancel these charges according to the Gauss law. Both problems can be fixed by introducing a so called orientifold projection.
To reduce the amount of supersymmetry, let us consider the worldsheet parity operator

$$\Omega : (\tau, \sigma) \mapsto (\tau, 2\pi - \sigma) \tag{2.3.31}$$

combined with an operator which gives us a sign depending on the left moving fermion number (F_L denotes the left moving fermion number operator):

$$\Omega(-1)^{F_L} \tag{2.3.32}$$

This acts on the Fock space of physical states by interchanging left/right moving sectors and giving a minus sign for an odd number of left moving fermions. Especially in the NS-R (R-NS)-sectors we see that only a linear combination of the two gravitinos is invariant under (2.3.32). In the type IIA case, which is the theory we always use for examples (but if not otherwise stated, similar arguments hold for the type IIB case):

$$\mathbf{8}_C \otimes \tilde{\psi}_{-\frac{1}{2}}|0, NS\rangle_R \quad + \quad \psi_{-\frac{1}{2}}|0, NS\rangle_L \otimes \tilde{\mathbf{8}}_R \tag{2.3.33}$$

This is invariant because we have physical states (and therefore $(-1)^{F_L} = 1$) and the world-sheet parity interchanges left movers and right movers. Therefore the projected theory has only $\mathcal{N} = 1$ supersymmetry.

If we also combine the above projection with an antiholomorphic involution $\bar{\sigma}$ of the internal space (if we can for example introduce complex coordinates $z_i, i = 1 \ldots 3$)

$$\bar{\sigma} : \mathcal{X} \to \mathcal{X}$$

$$z_i \mapsto \pm \bar{z}_i \qquad i = 1 \ldots 3 \tag{2.3.34}$$

we see that the fixed point set of the involution is a 3-cycle in the internal space and contains in addition the whole external Minkowski space:

$$\mathsf{Fix}(\bar{\sigma}) = \mathbf{R}^{1,3} \times \Sigma_3 \tag{2.3.35}$$

It turns out that this fixed point set also couples to the RR-fields. It is called orientifold plane, Op-plane for short, where p denotes the number of its spatial dimensions (similar to the notation for Dp-branes). The appropriate Chern-Simons-coupling has a similar structure as for the Dp-branes:

$$S_{O_p} = -Q_p \mu_p \int_{\mathbf{R}^{1,4} \times \Sigma_{p-3}} \sqrt{\frac{\mathfrak{L}(\mathcal{R}_T/4)}{\mathfrak{L}(\mathcal{R}_N/4)}} \wedge \bigoplus_q C_q \tag{2.3.36}$$

where $\mathcal{R}_T$ and $\mathcal{R}_N$ are the scalar curvatures of the tangent and normal bundle of the orientifold plane respectively and $\mathfrak{L}$ denotes the Hirzebruch-polynomial, which is given for reference in appendix 1. In addition Q_p denotes the charge of the orientifold plane which one can calculate via scattering amplitudes and in our normalisation convention has the value 2^{p-4}.

In the next section we want to combine the Chern-Simons-actions of D-branes and O-planes in such a way that the overall RR-charge in the internal space is zero.

2.3.1 Tadpole-cancellation conditions

In this section we want to derive a topological condition on the cycles wrapped by the D-branes and orientifold planes, which correspons physically to charge neutrality in the compact internal space. Consider orientifold p-planes (labelled by i) which contain the Minkowski space $\mathbf{R}^{1,3}$ ($p \geq 3$) and wrap $(p-3)$-cycles Σ_{p-3} in the internal space. In addition consider stacks (labelled by a) of N_a Dp-branes and their images under the orientifold map, which also contain the Minkowski space and wrap $(p-3)$-cycles in the internal Calabi-Yau. Firstly, we want to determine the equation of motion of the RR form C_{p+1}. The relevant kinetic part of the type IIA/B supergravity action is given by

$$S_{\text{kin}} = \frac{1}{8\kappa_{10}^2} \int_{M_{1,3} \times \mathcal{X}} F_{p+2} \wedge *F_{p+2} \qquad F_{p+2} = dC_{p+1} \tag{2.3.37}$$

where κ_{10} is proportional to the fourth power of the string length. In addition we use the Chern-Simons couplings given in (2.2.29) for the Dp-branes and in (2.3.36) for the Op-planes. To carry out the variation of the Chern-Simons couplings with respect to the RR form C_{p+1} we expand the square root expressions. For example, the first terms in the expansion for the Dp - brane case gives (using $p_1(T\Sigma_{p+1}) = p_1(TM_{1,3}) p_1(T\Sigma_{p-3})$)

$$\begin{aligned} \sqrt{\frac{\hat{\mathcal{A}}(R_T)}{\hat{\mathcal{A}}(R_N)}} = & \ (1 - \frac{1}{48} p_1(R_T^{(4)}) + \dots) \\ & \wedge \ (1 - \frac{1}{48} p_1(R_T^{\Sigma_{p-3}}) + \frac{1}{48} p_1(R_N^{\Sigma_{p-3}}) - \frac{1}{2 \cdot 24^2} p_1(R_T^{\mathcal{X}}) + \dots) \end{aligned} \tag{2.3.38}$$

and we get an expression of similar structure for the Op-planes ([18]). Expanding also the Chern character and then varying the relevant parts of the action with respect to C_{p+1} leads to

$$\delta_{C_{p+1}} S = \ \delta_{C_{p+1}} (S_{\text{kin}} + S_{Dp}^{CS} + S_{Op}^{CS})$$

$$= -\frac{1}{4\kappa_{10}^2}\int_{M_{1,3}\times\mathcal{X}} \delta C_{p+1}\wedge d * F_{p+2} + \mu_p \sum_a \int_{\Sigma_{Dp_a}} \text{ch}_0(F_a)\wedge \delta C_{p+1}$$
$$+ \mu_p \sum_{a'} \int_{\Sigma_{Dp_{a'}}} \text{ch}_0(F_a)\wedge \delta C_{p+1} - Q_p\mu_p \sum_i \int_{\Sigma_{Op_i}} \delta C_{p+1}. \qquad (2.3.39)$$

We observe that $\text{ch}_0(F) = \text{tr}(\mathbf{1}) = N_{D_p}$. If we also introduce the Poincaré-duals to the cycles Σ_{Dp} and Σ_{Op}

$$\pi_{Dp} := \text{PD}(\Sigma_{Dp}) \qquad \pi_{Op} := \text{PD}(\Sigma_{Op}) \qquad (2.3.40)$$

we get for the variation of the action

$$\delta_{C_{p+1}} S = -\frac{1}{4\kappa_{10}^2}\int_{M_{1,3}\times\mathcal{X}} \delta C_{p+1}\wedge *dF_{p+2} + \mu_p \sum_a N_{Dp_a} \int_{M_{1,3}\times\mathcal{X}} \delta C_{p+1}\wedge \pi_{Dp_a}$$
$$+ \mu_p \sum_{a'} N_{Dp_{a'}} \int_{M_{1,3}\times\mathcal{X}} \delta C_{p+1}\wedge \pi_{Dp_{a'}} - Q_p\mu_p \sum_i \int_{M_{1,3}\times\mathcal{X}} \delta C_{p+1}\wedge \pi_{Op_i}$$
$$(2.3.41)$$

which gives us the equation of motion

$$\frac{1}{4\kappa_{10}^2} d * F_{p+2} = \mu_p N_{Dp_a} \sum_a \pi_{Dp_a} + \mu_p N_{Dp_{a'}} \sum_{a'} \pi_{Dp_{a'}} - Q_p\mu_p \sum_i \pi_{Op_i} \qquad (2.3.42)$$

If we now consider the corresponding homology classes Π_{Dp}, $\Pi_{Op} \in H_3(\mathcal{X},\mathbb{Z})$ we can translate the above equation into homology. Observing that $d * F_{p+2}$ is exact, we get

$$\boxed{\sum_a N_a(\Pi_{Dp_a} + \Pi'_{Dp_a}) - Q_p \sum_i \Pi_{Op_i} = 0} \qquad (2.3.43)$$

which is also known as the *tadpole cancellation condition* with respect to the RR-form C_{p+1}.

Chapter 3

Intersecting D-brane models

In this chapter we consider type IIA string theory compactified on Calabi-Yau manifolds/orientifolds with D6-branes wrapping 3-cycles on the internal space. In general position (i.e. infinitesimal deformations do not change the geometrical properties) two internal 3-cycles intersect at points. As we have already seen, we get non-Abelian gauge groups at the intersections. It will turn out that the massless open string spectrum at the intersection will contain chiral fermions in four dimensions. The resulting anomalies are cancelled by the RR-tadpole cancellation condition described in the last chapter for the Abelian case and by the generalized Green-Schwarz-mechanism for the mixed Abelian non-Abelian and gravitational case. Thus this setup provides the possibility to construct consistent four-dimensional models of particle physics.

3.1 Branes intersecting at angles and chirality

To illustrate the appearance of chiral spinors, let us consider two D6-branes, $D6_a$ and $D6_b$ which both contain the four-dimensional Minkowski space. For simplicity let us take the six-dimensional internal space to be $\mathbb{C}^3$ with complex coordinates Z^1, Z^2, Z^3. Four-dimensional Minkowski space should be part of both branes and therefore the two branes wrap cycles of real dimension three in the internal space. Generically, they intersect at a point in the internal space and the intersection is characterized by three angles $\theta_1, \theta_2, \theta_3$, schematically depicted in figure 3.1.

Now consider the sector of open strings which have one endpoint on the first brane and one endpoint on the second. We are interested in the massless case and therefore both endpoints are confined to the intersection of the two branes. The only difference in the quantisation procedure are different boundary conditions on the

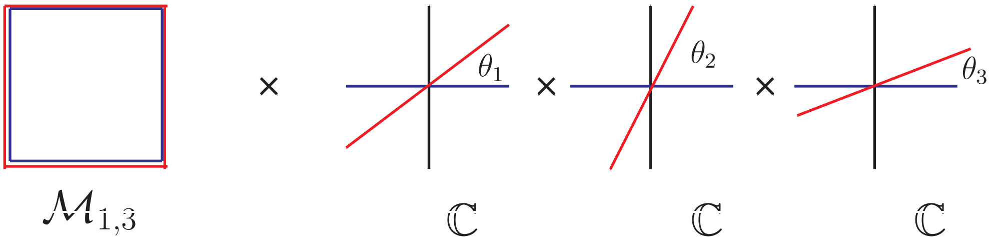


Figure 3.1: Two D6-branes containing the four-dimensional external space and wrapping different 3-cycles in the internal manifold

two endpoints, describing the confinement to the two different branes:

$$\begin{aligned} \sigma = 0: &\quad \partial_\sigma(Z^j + \bar{Z}^j) = \partial_\tau(Z^j - \bar{Z}^j) = 0 \\ \sigma = \pi: &\quad \partial_\sigma Z^j + e^{2i\theta_j}\partial_\sigma \bar{Z}^i = 0 \\ &\quad \partial_\tau Z^j - e^{2i\theta_j}\partial_\tau \bar{Z}^j = 0. \end{aligned} \tag{3.1.1}$$

These boundary conditions give rise to a modified mode expansion. In particular, the oscillator modes of both, world-sheet bosons and fermions get shifted by $\epsilon_j := \frac{\theta_j}{\pi}$:

$$\alpha_n^j \mapsto \alpha_{n+\epsilon_j}^j \quad \tilde{\alpha}_n^j \mapsto \tilde{\alpha}_{n-\epsilon_j}^j \qquad \text{(bosonic)} \tag{3.1.2}$$

$$\psi_{n+\nu}^j \mapsto \psi_{n+\nu+\epsilon_j} \quad \tilde{\psi}_{n+\nu}^j \mapsto \tilde{\psi}_{n+\nu-\epsilon_j}^j \qquad \text{(fermionic)} \tag{3.1.3}$$

where as usual $\nu = 0$ for the Ramond sector and $\nu = \frac{1}{2}$ for the Neveu-Schwarz sector. An important consequence of this shift is the following. In the generic case, where all θ_j are non-zero, all excitations in the Ramond-sector along the internal directions are massive. The only zero modes come from the directions along the common Minkowski space. These form, as in the unshifted case, a representation of the Clifford-algebra, but this time in four dimensions. The GSO-projection will project out one chirality and therefore the Ramond ground state is a single chiral Weyl fermion.

The mass of the lowest excitations of the NS-sector (which give scalars in four dimensions) depends on the θ_j. Only for specific values of them, the bosonic and fermionic degrees of freedom in four dimensions match. As a consequence supersymmetric models only arise (if at all) on intersecting brane configurations of specific values of the intersection angles.

3.2 Massless spectrum and family replication

Let us now consider stacks a, b of N_a and N_b D-branes respectively. They are assumed to wrap the 3-cycles

$$\Pi_a, \Pi_b \in H_3(\mathcal{X}, \mathbb{Z}) \tag{3.2.4}$$

which are in general not invariant under the orientifold projection $\Omega\bar{\sigma}(-1)^{F_L}$. For this case, the gauge symmetry realized on the intersection of the two cycles is $U(N_a) \times U(N_b)$. The degeneracy of the massless modes of the open string stretching between the two stacks is N_a and N_b respectively. We therefore see that the chiral fermion at the intersection described in the last section transforms under the bifundamental representation $(\Box_a, \bar{\Box}_b)$. If the string stretches between the stack a and its orientifold image a' the corresponding chiral fermion on the intersection of Π_a and its orientifold image Π'_a transforms in the symmetric $\Box\!\Box_a$ or antisymmetric $\begin{smallmatrix}\Box\\\Box\end{smallmatrix}_a$ representation of the gauge group $U(N_a)$.

In a compact internal space $\mathcal{X}$ 3-cycles generically have multiple intersections. Clearly one can create an arbitrary number of intersections by deforming the cycles wrapped by the brane. But the physically interesting quantity is the *net* number of chiral fermions which translates into the notion of topological intersection numbers. These are counted by the intersection product (M is a manifold of dimension n):

$$\circ : H_p(M, \mathbb{Z}) \times H_{n-p}(M, \mathbb{Z}) \to \mathbb{Z} \tag{3.2.5}$$

$$(\Pi_1, \Pi_2) \mapsto \int_M \eta_{\Pi_1} \wedge \eta_{\Pi_2} \tag{3.2.6}$$

where η_{Π_i} are the elements representing the cycles Π_i in cohomology.

Now we see that we get $\Pi_a \circ \Pi_b$ copies of the chiral fermionic matter arising at the intersection of the cycles Π_a and Π_b wrapped by stacks of D-branes. This is a geometrical explanation of the existence of multiple families of elementary particles. The exact rules ([19]) to compute the multiplicities of the various representations (depending on the type of object which wraps the cycles involved in the intersections) are given in the table.

Representation	Multiplicity
$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}_a$	$\frac{1}{2}(\Pi'_a \circ \Pi_a + \Pi_{O6} \circ \Pi_a)$
$\Box\!\Box_a$	$\frac{1}{2}(\Pi'_a \circ \Pi_a - \Pi_{O6} \circ \Pi_a)$
$(\Box_a, \bar{\Box}_b)$	$\Pi_a \circ \Pi_b$
$(\Box_a, \Box_b)$	$\Pi'_a \circ \Pi_b$

Table 3.1: Chiral matter for intersecting D6-brane models

3.3 Anomaly cancellation and Green-Schwarz mechanism

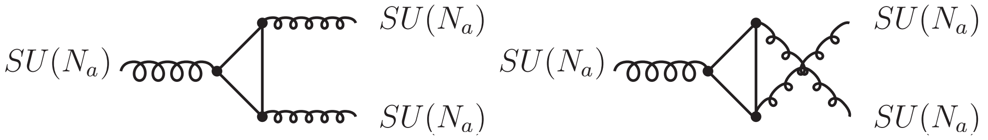
In the last section we described the existence of four-dimensional chiral fermions at the intersection of D-branes. As usual chiral fermions will be the source of gauge anomalies. In the following we will see that purely non-Abelian anomalies will cancel as a consequence of the RR-tadpole cancellation condition derived earlier. The case of Abelian, mixed Abelian non-Abelian and gravitational Anomalies will cancel due to the generalized version of the Green-Schwarz-mechanism.

3.3.1 Non-Abelian anomalies

To prove the absence of non-Abelian anomalies, we first recall the tadpole cancellation condition 2.3.43 for type IIA string theory with D6-branes and O6-planes:

$$\sum_a N_a(\Pi_{D_a} + \Pi'_{D_a}) - 4\sum_i \Pi_{O_i} = 0 \tag{3.3.7}$$

Now from field theory we know (e.g. [20]) that the non-Abelian anomaly coming from the graphs



is proportional to the group theory factor

$$\mathcal{A}^{aaa} \propto \mathrm{tr}(T^a_a\{T^b_a, T^c_a\}) = \frac{1}{2}A(r_a)d^{abc} \tag{3.3.8}$$

where r_a denotes the representation and T^k_a are the generators of the gauge group $SU(N_a)$. One can show that $A(r) = 1$ for the fundamental representation and $A(r) = N - 4$ ($A = N + 4$) for the antisymmetric (symmetric) representation of $SU(N)$ and $A(r) = -A(\bar{r})$ for the conjugate representation. Summing over all possible chiral fermions and using the multiplicities of table 3.1, we get for the anomaly

$$\begin{aligned}
\mathcal{A}^{aaa} \propto & \sum_{b\neq a} N_b(-\Pi_a \circ \Pi_b + \Pi'_a \circ \Pi_b) \\
& + \frac{N_a - 4}{2}(\Pi_a \circ \Pi'_a + \Pi_a \circ \Pi_{O6}) \\
& + \frac{N_a + 4}{2}(\Pi_a \circ \Pi'_a - \Pi_a \circ \Pi_{O6}) \\
& = \Pi_a \circ (\sum_{b\neq a} N_b(\Pi_b - \Pi'_b) + N_a\Pi_a) - 4\Pi_a \circ \Pi_{O6} \\
& = 0.
\end{aligned} \tag{3.3.9}$$

where we have used $\Pi_a \circ \Pi_a = 0$ for threecycles Π_a.

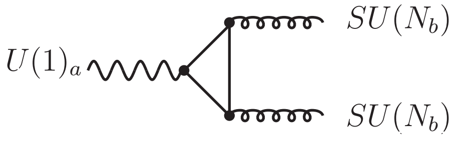
3.3.2 The Green-Schwarz mechanism

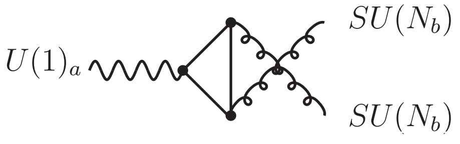
We now want to treat pure Abelian, mixed Abelian non-Abelian anomalies and gravitational anomalies. In the following we sketch the main arguments for the cancellation of these anomalies by introducing an appropriate Green-Schwarz-counterterm.

Let us first introduce useful objects for the later calculation. Let $\{\Gamma_i\}$ be a basis of $H^3(\mathcal{X},\mathbb{Z})$ and $\{\Lambda^j\}$ a dual basis w.r.t the intersection product ($\Gamma_i \circ \Lambda^j = \delta_i^j$). Therefore we can expand every 3-cycle $\Pi_a \in H^3(\mathcal{X},\mathbb{Z})$ as

$$\Pi_a = \alpha_a^i \Gamma_i = \sum_a \beta_{aj} \Lambda^j \tag{3.3.10}$$

Now we consider the $U(1)_a - SU(N_b)^2$ anomaly with contributions of the following type of diagrams:





They are proportional to the group theory factor ([20])

$$\mathcal{A}^{abb} \propto \sum_r Q_a(r) C_b(r) \tag{3.3.11}$$

where the sum goes over the representations of chiral fermions in the loop, Q_a is the $U(1)$-charge of stack a and $C_b(r)$ is $\frac{1}{2}$ for the fumdamental/antifundamental and $\frac{N+2}{2}$ $(\frac{N-2}{2})$ for the symmetric (antisymmetric) representations. Writing this again with the help of the multiplicities given in table 3.1 we get

$$\begin{aligned}\mathcal{A}^{abb} \propto & \frac{1}{2}\delta_{ab}(\sum_l N_{\Pi_l}(\Pi_l + \Pi_l') - 4\sum_l \Pi_{O6_l}) \circ \Pi_b + \frac{N_a}{2}(-\Pi_a + \Pi_a') \circ \Pi_b \\ = & \frac{N_a}{2}(-\Pi_a + \Pi_a') \circ \Pi_b\end{aligned} \tag{3.3.12}$$

where we have again used the tadpole cancellation condition 2.3.43. In the following, we want to use the Chern-Simons couplings of the RR-forms to the stack a to show that by dimensional reduction we get couplings which cancel the above field theoretic anomaly. Thus string theory provides a mechanism to get consistent models of chiral gauge theories in four dimensions. Consider the following Chern-Simons couplings of the RR-forms C_3 and C_5 to the field strength F_a on the stack a:

$$\int_{R^{1,3}\times\Pi_a} C_3 \wedge \mathrm{tr}(F_a \wedge F_a) \qquad \int_{R^{1,3}\times\Pi_a} C_5 \wedge \mathrm{tr}F_a. \tag{3.3.13}$$

We now want to perform a dimensional reduction. Therefore we use the decomposition 3.3.10 and integrate over the internal parts to obtain the following four-dimensional scalars (axions) and two-forms:

$$\Phi^i = \int_{\Gamma_i} C_3 \qquad B_j = \int_{\Lambda^j} C_5 \tag{3.3.14}$$

using them we can write the couplings 3.3.13 as follows:

$$\sum_i \alpha_a^i \int_{R^{1,3}} \Phi_i \mathrm{tr} F_a \wedge F_a; \qquad N_a \sum_j \beta_j \int_{R^{1,3}} B_2^j \wedge F_a \tag{3.3.15}$$

where the factor N_a comes from the trace over the $U(1)$-generator if we decompose $U(N_a) = SU(N_a) \times U(1)$. These couplings can be used to get the following Green-Schwarz-diagram

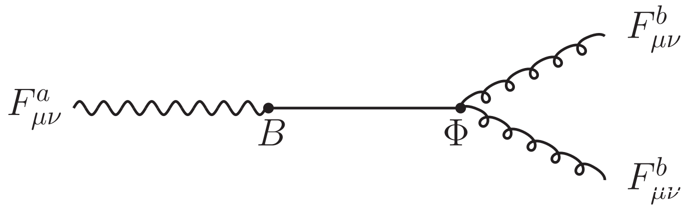


Figure 3.2: Tree-level axionic contribution

The amplitude will be proportional to the vertex coefficients, which we get from 3.3.15:

$$\begin{aligned} \mathcal{A}_{GS} &\propto \frac{N_a}{2} \sum_i (\alpha_a^i - \alpha_a^{i\prime}) \beta_{bi} \\ &= \frac{N_a}{2} \sum_{i,j} (\alpha_a^i - \alpha_a^{i\prime}) \beta_{bj} \Gamma_i \circ \Lambda^j \\ &= \frac{N_a}{2} (\Pi_a - \Pi_a') \circ \Pi_b \end{aligned} \tag{3.3.16}$$

We get the $\frac{1}{2}$-factor from the symmetry of the diagram and therefore we see that it will cancel the field theoretic anomaly 3.3.12.

Similar cancellations can be observed for the other types of anomalies, like the cubic Abelian and gravitational cases ([14]).

An additional important observation of the above couplings 3.3.15 is the fact that some of the Abelian gauge fields get massive via a Stückelberg-type mechanism. Therefore they are still global symmetries at the perturbative level and therefore restrict the form of allowed couplings in the theory. This will be discussed in the next section.

3.3.3 $U(1)$-symmetries

We again recall the schematic Green-Schwarz coupling structure of the $U(1)$-gauge field F_a on the stack a to the two-form B

$$\int_{\mathbb{R}^{1,3}} m\ B^j \wedge F_a \tag{3.3.17}$$

Consider now the following Lagrangian, containing in addition the kinetic terms for the 2-form B and for the $U(1)$-gauge field F_a:

$$\mathcal{L} = -dB \wedge *dB - \frac{1}{4g^2} F_a \wedge *F_a + mB \wedge F_a \tag{3.3.18}$$

We want to show that such a Lagrangian is equivalent to a Lagrangian with a mass term for the gauge field A_a or some gauge equivalent configuration, with $F_a = dA_a$. To show this we observe that the above Lagrangian can be reformulated with an independent field H plus the constraint that $dH = 0$ (and therefore locally $H = dB$) which is implemented by an auxiliary field a:

$$\mathcal{L} = -H \wedge *H - \frac{1}{4g^2} F_a \wedge *F_a + mB \wedge dA_a + m\ a\ dH \tag{3.3.19}$$

if we do a partial integration in the last two terms we get (assuming that the boundary terms vanish)

$$\mathcal{L} = -H \wedge *H - \frac{1}{4g^2} F_a \wedge *F_a - m\ H \wedge (A_a + da) \tag{3.3.20}$$

Now using the resulting equation of motion $*H = -\frac{m}{2}(A_a + da)$ we get

$$\mathcal{L} = -\frac{m^2}{4}(A_a + da) \wedge *(A_a + da) - \frac{1}{4g^2} F \wedge *F \tag{3.3.21}$$

which gives a mass to the field $A_a + da$. This is also called the Stückelberg mechanism. To preserve gauge invariance this mechanism also demands that the scalar a transforms under $U(1)$ gauge transformations to compensate the transformation of the gauge field A_a. As a result, $U(1)$-symmetries which become massive in this way will still be symmetries in perturbation theory and therefore will often forbid phenomenologically desired couplings later on. This will be one of the reasons for the introduction of D-brane instantons. Because we also want at least one massless $U(1)$ to have electromagnetism, we are finally interested in the conditions under which a linear combination of $U(1)'s$ will remain massless. Recall the coupling which generates the Stückelberg-mass:

$$\int_{\mathbb{R}^{1,3}} N_a(\pi_{aj} - \pi'_{aj}) B^j \wedge F_a \tag{3.3.22}$$

If we now consider a set of stacks of D6-branes labelled by x with N_x branes at the stack x we can look at the linear combination $q_x F_x$ of the $U(1)$-field strengths. The Stückelberg coupling then becomes

$$\sum_x \int_{\mathbb{R}^{1,3}} N_x(\pi_{xj} - \pi'_{xj}) B^j \wedge q_x F_x \tag{3.3.23}$$

and therefore the condition for a linear combination of $U(1)$'s to remain massless can be read off:

$$\sum_x q_x N_x(\pi_x - \pi'_x) = 0 \tag{3.3.24}$$

This is again a topological condition which will play an important restrictive role in finding realistic models of particle physics.

Chapter 4

D-brane instantons

4.1 Instantons in Yang-Mills theories and collective coordinates

In the first section we want to briefly recall the basic elements of Yang-Mills instantons ([21][22][23][26]). We first want to state an instanton-solution to $SU(2)$ Yang-Mills theory and then describe the behaviour of path integrals if we expand the physical fields around an instanton solution. It will turn out that it is necessary to integrate exactly over the collective coordinates of the instanton solution to avoid an infinite path integral.

4.1.1 Finite action solutions by selfdual/anti-selfdual connections

Consider the pure Euclidean Yang-Mills action over a compact manifold M

$$\mathcal{L}_{YM} = \frac{1}{g^2} \int_M \operatorname{tr}(F \wedge *F) \tag{4.1.1}$$

where the trace is over the adjoint representation of the gauge group. To write this more conveniently we use the scalar product of forms η, ζ with values in the Lie algebra $\mathfrak{g}$ of the gauge group G:

$$\langle \eta \otimes X, \zeta \otimes Y \rangle := \eta \wedge *\zeta \otimes \operatorname{tr}(\mathrm{ad}X \circ \mathrm{ad}Y) \tag{4.1.2}$$

where $\eta, \zeta \in \wedge^k T^* M$, $X, Y \in \mathfrak{g}$ and ad denotes the adjoint representation. With this definition we get

$$\mathcal{L}_{YM} = \frac{1}{g^2} \int_M \langle F, F \rangle \tag{4.1.3}$$

Observing that the Hodge-star on 2-forms squares to the identity in four Euclidean dimensions, we can estimate the Yang-Mills action from below:

$$\begin{aligned}0 &\leq \langle F \pm *F, F \pm *F\rangle \\ &= \langle F, F\rangle \pm \langle *F, F\rangle \pm \langle F, *F\rangle + \langle *F, *F\rangle \\ &= 2(\langle F, F\rangle \pm \langle *F, F\rangle)\end{aligned} \tag{4.1.4}$$

and therefore we get a lower bound for the Yang-Mills-action:

$$S_{YM} \geq | \int_M \mathrm{tr} F \wedge F \ | \tag{4.1.5}$$

And we observe that this bound is reached by selfdual or anti-selfdual field strengths. Therefore to minimize the action we only have to solve the (anti-)selfduality condition. This was first done in [22] for the gauge group $SU(2)$: If we want to get a solution to the Yang-Mills equations for the connection A ($F = D^A A$, D^A the gauge covariant derivative), the field strengths should vanish for $|x| \to \infty$. Therefore the following ansatz is reasonable:

$$A = f(r)U^{-1}dU \qquad U : \mathbb{R}^4 \backslash \{0\} \to SU(2) \tag{4.1.6}$$

Here, f is a real function of $r = |x|$ which has the property $\lim_{r\to\infty} f(r) = 1$ in order to get a pure gauge ansatz. The (anti-)selfduality conditions then lead to ordinary differential equations for f and one can show that the solution in the so called singular gauge [26] is given by ($\bar{\eta}^a_{\mu\nu}$ are the t'Hooft symbols defined in the appendix):

$$A^a_\mu(x; x_0; \rho) = 2\frac{\rho^2 \bar{\eta}^a_{\mu\nu}(x - x_0)^\nu}{(x - x_0)^2[(x - x_0)^2 + \rho^2]} \tag{4.1.7}$$

We observe that the solution is characterized by 4 parameters x_0 (the position of the instanton) and ρ (the size of the instanton) which are called the collective coordinates or moduli of the instanton. We will see later that they will play an important role in the evaluation of the path integral in the presence of an instanton.

The class of maps $U : \mathbb{R}^4 \backslash \{0\} \to SU(2)$ is topologically the same as the maps $S^3 \to S^3$ and therefore is classified by the homotopy group $\pi_3(S^3) \cong \mathbb{Z}$ and one can show that for every $n \in \mathbb{Z}$ there is a solution minimizing the action with the minimal value

$$S_{YM} = \frac{1}{g^2} \int_M \mathrm{tr} F \wedge F = n\frac{8\pi^2}{g^2} \tag{4.1.8}$$

where $n \in \mathbb{Z}$ is usually referred to as the *instanton number.* Atiyah, Drinfield, Hitchin and Manin gave a construction of instanton solutions to all numbers $n \in \mathbb{Z}$ ([24]) and it was given for more general gauge groups in [25].

4.1.2 The path integral in the presence of an instanton and collective coordinates

We now want to calculate correlation functions in an instanton background by expanding the action around the classical instanton solution.

Bosonic collective coordinates

Let us first consider a system with bosonic fields ϕ^j and action $S[\phi^j]$. We decompose the fields to treat quantum fluctuations around the classical instanton solution:

$$\phi^j(x) = \phi^j_{cl}(x,\lambda) + \phi^j_{qu}(x,\lambda) \tag{4.1.9}$$

where we have written down explicitly the dependence of the classical instanton solution and the quantum part on the collective coordinates as seen in the last section. The collective coordinates only enter by introducing the split into the classical and quantum part. Let us also expand the action up to second order in the fields:

$$S[\phi] = S_{cl} + \frac{1}{2}\phi^i_{qu}\mathfrak{M}_{ij}\phi^j_{qu} \qquad \mathfrak{M}_{ij} = \frac{\delta^2 S_{cl}[\phi]}{\delta\phi^i(x,\lambda)\delta\phi^j(x,\lambda)} \tag{4.1.10}$$

Where we denote for convenience the operator $\mathfrak{M}$ and its integral kernel representation by the same letter. We now observe that the operator $\mathfrak{M}$ has zero modes, which are by definition nonzero, normalizable eigenfunctions of $\mathfrak{M}$ with zero eigenvalue. They are given by $\zeta^j := \frac{\partial\phi^j_{cl}}{\partial\lambda}$. To prove this, we take the derivative of the equations of motion w.r.t. the collective coordinate λ and assume the normalizability:

$$\begin{aligned} 0 &= \frac{\partial}{\partial\lambda}\frac{\delta S_{cl}}{\delta\phi^j_{cl}(x,\lambda)} \\ &= \int dy \frac{\delta^2 S_{cl}[\phi]}{\delta\phi^i_{cl}(x,\lambda)\delta\phi^j_{cl}(y,\lambda)}\frac{\delta\phi^j_{cl}(y,\lambda)}{\partial\lambda} = \mathfrak{M}_{ij}\frac{\partial\phi^j_{cl}}{\partial\lambda} \end{aligned} \tag{4.1.11}$$

If now $\mathfrak{M}$ is a self adjoint operator, we can expand the quantum fluctuations into eigenfunctions E^j of $\mathfrak{M}$:

$$\mathfrak{M}_{ij}E^j_\kappa = \epsilon_\kappa E^j_\kappa; \quad \phi^j_{qu} = \sum_{m\in\mathbb{N}} \xi_m E^j_m; \quad \langle E_m, E_n\rangle = c_m\delta_{mn} \tag{4.1.12}$$

Then we can write the path integral as

$$\int [d\phi]e^{-S[\phi]} = \int (\prod_{m\in\mathbb{N}} \sqrt{c_m}d\xi_m)e^{-S_{cl}+\frac{1}{2}\sum_{m\in\mathbb{N}}\xi^2_m\epsilon_m c_m} \tag{4.1.13}$$

If there are no zero modes, the Gaussian integration would lead to a result proportional to $(\det\mathfrak{M})^{-\frac{1}{2}}$, but in the presence of zero modes this would be infinite and thus the path integral ill-defined. We therefore single out the integration over the zero mode to get

$$\int [d\phi]e^{-S[\phi]} = \int d\xi_0\sqrt{c_0}e^{-S_{cl}}(\det{}'\mathfrak{M})^{-\frac{1}{2}} \tag{4.1.14}$$

where $\det'\mathfrak{M}$ comes from performing the Gaussian integration without the zero mode ξ_0 (which is also called the amputated determinant).

Now to convert this into an integration over the collective coordinates, we do a Faddeev-Popov-insertion:

$$1 = \int d\lambda \delta(g(\lambda)) \frac{\partial g(\lambda)}{\partial \lambda}. \tag{4.1.15}$$

We will do this insertion with $g(\lambda) = -\langle \phi - \phi_{cl}(\lambda) | \frac{\partial \phi_{cl}}{\partial \lambda} \rangle$ and therefore we get:

$$\begin{aligned} \int [d\phi] e^{-S[\phi]} &= \int d\lambda d\xi_0 \sqrt{c_0} e^{-S_{cl}} (\det{}'\mathfrak{M})^{-\frac{1}{2}} \delta(-\langle \phi_{qu} | \frac{\partial \phi_{cl}}{\partial \lambda} \rangle)(-\frac{\partial}{\partial \lambda} \langle \phi_{qu}(\lambda) | \frac{\partial \phi_{cl}}{\partial \lambda} \rangle) \\ &= \int d\lambda d\xi_0 \sqrt{c_0} e^{-S_{cl}} (\det{}'\mathfrak{M})^{-\frac{1}{2}} \delta(\xi_0 c_0)(c_0 - \langle \phi_{qu}(\lambda) | \frac{\partial E}{\partial \lambda} \rangle) \\ &= \int d\lambda \sqrt{c_0} e^{-S_{cl}} (\det{}'\mathfrak{M})^{-\frac{1}{2}} \end{aligned} \tag{4.1.16}$$

here we have neglected the term $\langle \phi_{qu}(\lambda) | \frac{\partial E}{\partial \lambda} \rangle$ because in a semiclassical approximation the solution ϕ_{cl} should not oscillate too strong. For more zero modes, we get ([26])

$$\int [d\phi] e^{-S} = \int \prod_i d\lambda_i (\det U)^{\frac{1}{2}} e^{-S_{cl}} (\det{}'\mathfrak{M})^{-\frac{1}{2}} \tag{4.1.17}$$

where U denotes the metric on the space of collective coordinates.

Fermionic collective coordinates

Again we use the ansatz $\phi(x) = \phi_{cl}(x, \lambda) + \phi_{qu}(x, \lambda)$ and we can expand the quantum fields ϕ_{qu} in eigenfunctions

$$\phi^j_{qu} = \sum_{m \in \mathbb{N}} \xi_m E^j_m \tag{4.1.18}$$

but now the ξ_m are Grassmann-valued parameters. Using again the definition (4.1.13) of the path integral measure we now get because of the Berezin-integration-rules:

$$\int [d\phi] e^{-S[\phi]} = \int d\xi_0 \sqrt{c_0} e^{-S_{cl}} (\det{}'\mathfrak{M})^{\frac{1}{2}} \tag{4.1.19}$$

and after transforming the integration variable ξ_0 into the collective coordinates of the fermionic instanton solution ([26]) one again has an integration over the Grassmann-valued (fermionic) collective coordinates.

4.2 D-brane instantons

In this section we now turn to instantonic objects in string theory. Here we will come to a generalization of field theory instantons. A *Dp-brane instanton* is by definition an object which is pointlike in the external space and wraps a $(p+1)$-dimensional cycle in the internal Calabi-Yau space. They are often called Euclidean branes or Euclidean instantons because they are only extended in the (Euclidean) internal dimensions. Our first example are the D(-1) branes which are simply points in the external space. Placing them into spacetime filling D3-branes, we will be able to derive the ADHM-constraints and the BPST instanton solution from string scattering amplitudes. Therefore it is reasonable to consider those objects as string theoretic realization of gauge instantons.

4.2.1 The ADHM-constraints from string scattering

We now want to describe how the ADHM constraints arise from string scattering amplitudes in the $\alpha' \to 0$ limit and how one gets the classical instanton solution stated in chapter 4.1 . It was realized in [28] that one can describe an instanton solution with topological charge k in $SU(N)$ gauge theory if we place k $D(-1)$ branes into a stack of N D3-branes (which fill out the four dimensional external space). In order to find the effective Lagrangian of the massless fields on the D3 branes, we consider scattering amplitudes of the involved fields and perform the limit $\alpha' \to 0$ where the Yang Mills coupling remains constant (this is often called the field theory limit). As an example, we get the interaction term of a gauge boson A^μ and two gauginos $\Lambda^{\alpha A}$ by calculating the following disc-amplitude

$$\mathcal{M}_{\bar{\Lambda}A\Lambda} \propto \langle V_{\bar{\Lambda}}^{(-\frac{1}{2})}(z_1) V_A^{(-1)}(z_2) V_\Lambda^{(-\frac{1}{2})}(z_3)\rangle z_{12}\ z_{23}\ z_{31} \tag{4.2.20}$$

where $z_{ij} = z_i - z_j$ and the superscripts on the vertex operators denote the ghost charges. The vertex operators needed for this amplitude are given by (we first drop factors of $2\pi\alpha'$ and just give the dependence in the final results by reintroducing them in the following way: We have $(2\pi\alpha')^{\frac{1}{2}}$ for bosonic fields of the NS sector and $(2\pi\alpha')^{\frac{3}{4}}$ for the fermionic fields of the R sector)

$$V_A^{(-1)} = A^\mu \frac{1}{\sqrt{2}} \psi_\mu(z) e^{-\phi(z)} e^{ip_\mu X^\mu(z)} \tag{4.2.21}$$

$$V_{\bar{\Lambda}}^{(-\frac{1}{2})} = \bar{\Lambda}_{\dot{\alpha}A} S^{\dot{\alpha}}(z) S^A(z) e^{-\frac{1}{2}\phi(z)} e^{ip_\mu X^\mu} \tag{4.2.22}$$

$$V_\Lambda^{(-\frac{1}{2})} = \Lambda_{\alpha A} S_\alpha(z) S_A(z) e^{-\frac{1}{2}\phi(z)} e^{ip_\mu X^\mu} \tag{4.2.23}$$

Here we assume Euclidean signature. X, ψ denote the string coordinates and S the spin fields. We use $\mu, \nu = 1 \ldots 4$ for the external dimensions, $a, b = 5 \ldots 10$ for the internal dimensions and $M, N = 1 \ldots 10$ for complete space. A ten dimensional spin field decomposes into Weyl spinors $S_\alpha (S^{\dot{\alpha}})$of $SO(4)$ of positive (negative) chirality and Weyl spinors $S^A (S_A)$ of $SO(6)$ of positive (negative) chirality. Inserting this into 4.2.20 we can factorize the different sectors which leads to the evaluation of

$$\begin{aligned}
&\bar{\Lambda}_{\dot{\alpha}A} \langle S^{\dot{\alpha}}(z_1) S^A(z_1) \psi_\mu(z_2) S_\alpha(z_3) S_B(z_3)\rangle \Lambda^{\alpha B}(z_3) \frac{1}{\sqrt{2}} A_\mu(p) \\
&\langle e^{-\frac{1}{2}\phi(z_1)} e^{-\phi(z_2)} e^{-\frac{1}{2}\phi(z_3)}\rangle \langle e^{ip_\mu^1 X^\mu(z_1)} e^{ip_\mu^2 X^\mu(z_2)} e^{ip_\mu^3 X^\mu(z_3)}\rangle z_{12}\ z_{23}\ z_{31} \\
&= -\frac{1}{\sqrt{2}} \mathrm{C} \bar{\Lambda}_{\dot{\alpha}A} \bar{\sigma}_\mu{}^{\dot{\alpha}}{}_\beta A_\mu(p) \Lambda^{\beta B} \delta^A{}_B
\end{aligned} \tag{4.2.24}$$

Where we used the correlators given in the appendix and C is a normalisation constant. Taking the trace over the gauge group indices, summing over the different orderings of the generators of the gauge group (for the rules of computing disc diagrams see for example [11]) and writing out the normalisation factor C, we arrive at

$$-\frac{2i}{g_{YM}^2}\mathrm{tr}(\bar{\Lambda}_{\dot{\alpha}A}[\not{A}^{\dot{\alpha}\beta},\Lambda_{\beta}^{A}]) \qquad (4.2.25)$$

where we get a factor of $(2\pi\alpha')^2$ from the fields and in addition we introduced a normalisation constant for disc amplitudes for D3-branes which is given by $\frac{1}{\pi^2\alpha'^2 g_{YM}^2}$ (see for example [33]). We can proceed in the same way for all the massless fields of the D3-D3 sector. Taking the $\alpha' \to 0$ limit (with the Yang-Mills coupling fixed) and doing a Fourier transform one can derive the Euclidean $\mathcal{N} = 4$ super-Yang-Mills action which is responsible for these interactions ([30]):

$$\begin{aligned} S =& \frac{1}{g_{YM}^2}\int d^4x \ \mathrm{tr}\{\frac{1}{2}F_{\mu\nu}^2 - 2\bar{\Lambda}_{\dot{\alpha}A}\bar{\not{D}}^{\dot{\alpha}\beta}\Lambda_{\beta}^{A} + (D_\mu\phi_a)^2 \\ & -\frac{1}{2}[\phi_a,\phi_b] - i(\Sigma^a)^{AB}\bar{\Lambda}_{\dot{\alpha}A}[\phi_a,\bar{\Lambda}_{B}^{\dot{\alpha}}] - i(\bar{\Sigma}^a)_{AB}\Lambda^{\alpha A}[\phi_a,\Lambda_{\alpha}^{B}]\}. \end{aligned} \qquad (4.2.26)$$

But in addition to that we can also consider the massless fields in the D(−1)-D(−1) sector and in the D3-D(−1)-sector. We first give the relevant fields and their properties:

sector	modulus	vertex operator	representation
NS	a_μ	$\frac{a_\mu}{\sqrt{2}}\psi^\mu(z)e^{-\phi(z)}$	adoint of $U(k)$
NS	χ_m	$\frac{1}{\sqrt{2}}\chi_m\psi_m(z)e^{-\phi(z)}$	”
NS	D_c	$\frac{1}{2}D_c\bar{\eta}_{\mu\nu}^{c}\psi^\mu(z)\psi^\nu(z)$	”
R	$M^{\alpha A}$	$M^{\alpha A}S_\alpha(z)S_A z e^{-\frac{1}{2}\phi(z)}$	”
R	$\lambda_{\dot{\alpha}A}$	$\lambda_{\dot{\alpha}A}S^{\dot{\alpha}}(z)S^A(z)e^{-\frac{1}{2}\phi(z)}$	”

Table 4.1: D(-1)-D(-1)-sector

sector	modulus	vertex operator	representation
NS	$w_{\dot{\alpha}}$	$w_{\dot{\alpha}}\Delta(z)S^{\dot{\alpha}}(z)e^{-\phi(z)}$	$(\mathbf{k},\mathbf{N})$
NS	$\bar{w}^{\dot{\alpha}}$	$\bar{w}^{\dot{\alpha}}\bar{\Delta}(z)S_{\dot{\alpha}}(z)e^{-\phi(z)}$	$(\mathbf{N},\mathbf{k})$
R	μ_A	$\mu_A\Delta(z)S^A(z)e^{-\frac{1}{2}\phi(z)}$	$(\mathbf{k},\mathbf{N})$
R	$\bar{\mu}_A$	$\bar{\mu}_A\bar{\Delta}(z)S_A(z)e^{-\frac{1}{2}\phi(z)}$	$(\mathbf{N},\mathbf{k})$

Table 4.2: D(−1) - D3 - sector

Δ and $\bar{\Delta}$ denote the twist fields which change the boundary conditions from the $D(3)$ to the $D(-1)$ sector and vice versa.

Now again the scattering amplitudes will give us the relevant information about the action of the moduli fields. Let us, for example, consider the following amplitude

$$\begin{aligned}\mathcal{A}_{\lambda a M} \;\propto\;& \operatorname{tr}\{z_{12}z_{23}z_{31}\lambda_{\dot\alpha A}\frac{1}{\sqrt{2}}\langle S^{\dot\alpha}(z_1)S^A(z_1)\psi_\mu(z_2)S_\beta(z_3)S_B(z_3)\rangle \\ & a^\mu M^{\beta B}\langle e^{-\frac{1}{2}\phi(z_1)}e^{-\phi(z_2)}e^{-\frac{1}{2}\phi(z_3)}\rangle\} \\ =& - i\ \operatorname{tr}\{\lambda_{\dot\alpha A}(\bar\sigma_\mu)^{\dot\alpha}_{\beta}a^\mu M^{\beta A}\}\end{aligned} \tag{4.2.27}$$

which will give a contribution of the form

$$\mathcal{A}_{\lambda a M} = -\frac{i}{g_0^2}\operatorname{tr}(\lambda_{\dot\alpha A}[\not{a}^{\dot\alpha\beta}, M^A_\beta]) \tag{4.2.28}$$

Where we have introduced again the constants appropriate to the type of fields (as explained above) which give us $(2\pi\alpha')^2$ and in addition we have a factor of $\frac{1}{4\pi^2\alpha'^2 g_0^2}$ which together give only a dependence of g_0. This is the analogue of the Yang-Mills coupling for the D(-1) branes and is kept constant in the field theory limit . If we calculate in a similar way all the 3-point correlators in the D(-1)-D(-1) and D3-D(-1) sector we arrive at the action for the moduli given by [30]

$$\begin{aligned}S_m = \operatorname{tr}\{ &- [a_\mu, \chi^m]^2 - \frac{i}{4}M^{\alpha A}[\chi_{AB}, M^b_\alpha] + \chi^m \bar w_{\dot\alpha} w^{\dot\alpha}\chi_m + \frac{i}{2}\bar\mu^A\mu^B\chi_{AB} \\ &- iD^c(\bar w^{\dot\alpha}(\tau^c)^{\dot\beta}_{\dot\alpha}w_{\dot\beta} + i\bar\eta^c_{\mu\nu}[a^\mu, a^\nu]) \\ &+ i\lambda^{\dot\alpha}_A(\bar\mu^A w_{\dot\alpha} + \bar w_{\dot\alpha}\mu^A + \sigma^\mu_{\beta\dot\alpha}[M^{\beta A}, a_\mu])\}\end{aligned} \tag{4.2.29}$$

where $\chi_{AB} = \chi_m(\Sigma^m)_{AB}$.

We observe that the equations of motion for D^c and $\lambda^{\dot\alpha}_A$ exactly correspond to the ADHM constraints (given in [26]).

4.2.2 The BPST instanton from string scattering

It is also possible to get the instanton solution in classical field theory. For simplicity we consider only the case of topological charge 1, i.e. we have an Abelian Chan-Paton gauge group in the (-1)-sector. In addition we restrict to $SU(2)$ gauge theory on the D3-branes to get the original BPST solution. We have to evaluate the disc amplitude with two moduli and one gauge field inserted given in figure 4.1.

To calculate the amplitude we use the vertex operators for the moduli w and $\bar w$ of the above tables. They are in the (-1)-ghost picture so we have to chose the vertex operator for the gauge field in the zero ghost picture. Applying the picture changing operation we get:

$$A^{(0)I}_\mu = 2iT^I(\partial X_\mu - ip\cdot\psi\psi_\mu)e^{-ipX} \tag{4.2.30}$$

and thus we get for the amplitude

$$\mathcal{M}_{\bar w A w} = \langle V^{(-1)}_{\bar w} V_{A^I_\mu(p)} V^{(-1)}_w\rangle z_{12}z_{23}z_{31}$$

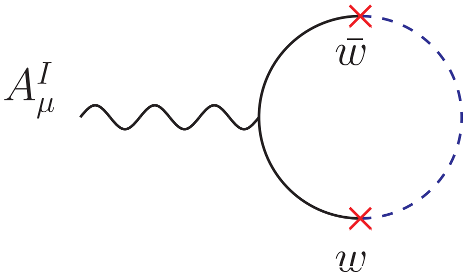


Figure 4.1: Disc diagram leading to the BPST instanton solution: The two insertions of the moduli vertex operators change the boundary from the D3 sector (which is represented by a solid line) to the D(-1) sector (represented by a dashed line).

$$\begin{aligned}
&= \bar{w}_{\dot{\alpha}} \langle \bar{\Delta}(z_1) e^{-ipX(z_1)} \Delta(z_3) \rangle w_{\dot{\beta}} z_{12} z_{23} z_{31} \langle e^{-\phi(z_1)} e^{-\phi(z_3)} \rangle \\
&\quad \langle S^{\dot{\alpha}}(z_1) 2 i T^I (\partial X_\mu - ip \cdot \psi \psi_\mu(z_2) S^{\dot{\beta}}(z_3) \rangle \\
&= i (T^I)^a_b p^\nu \bar{\eta}^c_{\mu\nu} (w_{\dot{\alpha}}{}^b (\tau_c)^{\dot{\alpha}}{}_{\dot{\beta}} \bar{w}^{\dot{\beta}}{}_a) e^{-ipx_0}
\end{aligned} \tag{4.2.31}$$

where we have used the correlators and t'Hooft symbols $\eta^c_{\mu\nu}$ given in the appendix.

Now if we want to calculate the gauge potential $A_\mu(x)$ in position space we have to multiply the above amplitude with the propagator $\delta_{\mu\nu}/p^2$ and then take the Fourier transform with respect to the momentum variable p:

$$\begin{aligned}
A^I_\mu(x) &= \int \frac{d^4p}{2\pi^2} A^I_\mu(p, \bar{w}, w) \frac{1}{p^2} e^{ipx} \\
&= \, -2 (T^I)^a{}_b (w_{\dot{\alpha}}{}^b (\tau_c)^{\dot{\alpha}}{}_{\dot{\beta}} \bar{w}^{\dot{\beta}}{}_a) \bar{\eta}^c_{\mu\nu} \frac{(x-x_0)^\nu}{(x-x_0)^4}
\end{aligned} \tag{4.2.32}$$

To write this solution more conveniently, let us define $2\rho^2 := \bar{w}^{\dot{\alpha}}{}_a w_{\dot{\alpha}}{}^a$ and the following matrices

$$\mathfrak{t}_c{}^a{}_b = \frac{1}{2\rho^2} (w_{\dot{\alpha}}{}^a (\tau_c)^{\dot{\alpha}}{}_{\dot{\beta}} \bar{w}^{\dot{\beta}}{}_b) \tag{4.2.33}$$

Now if we look at the ADHM constraints in the case of topological charge 1 given in equation 4.2.29 which in this case reads

$$\bar{w}^{\dot{\alpha}} (\tau_c)^{\dot{\beta}}{}_{\dot{\alpha}} w_{\dot{\beta}} = 0 \tag{4.2.34}$$

With the help of this constraint we get the following

Claim: The $\mathfrak{t}$-matrices obey the same algebra as the generators of the $SU(2)$ gauge theory.

Proof: We have to show that

$$\mathfrak{t}_i{}^a{}_b \mathfrak{t}_j{}^b{}_c - \mathfrak{t}_j{}^a{}_b \mathfrak{t}_i{}^b{}_c = i \epsilon_{ijk} \mathfrak{t}_k{}^a{}_c \tag{4.2.35}$$

The left hand side is given by

$$\frac{1}{4\rho^4}[w_{\dot{\alpha}}{}^{a}(\tau_i)^{\dot{\alpha}}{}_{\dot{\beta}}\bar{w}^{\dot{\beta}}{}_{b}w_{\dot{\gamma}}{}^{b}(\tau_j)^{\dot{\gamma}}{}_{\dot{\delta}}\bar{w}^{\dot{\delta}}{}_{c} - w_{\dot{\alpha}}{}^{a}(\tau_j)^{\dot{\alpha}}{}_{\dot{\beta}}\bar{w}^{\dot{\beta}}{}_{b}w_{\dot{\gamma}}{}^{b}(\tau_i)^{\dot{\gamma}}{}_{\dot{\delta}}\bar{w}^{\dot{\delta}}{}_{c}]. \tag{4.2.36}$$

Now we use the fact that the identity and the Pauli matrices form a basis of complex 2-dimensinal matrices:

$$\bar{w}_{\dot{\beta}}{}^{b}w^{\dot{\gamma}}{}_{b} = 2\rho^2\mathbf{1} + m_i(\tau^i)^{\dot{\beta}}{}_{\dot{\gamma}} \tag{4.2.37}$$

Inserting this in the left hand side of the above equation, we see that the part multiplying the identity exactly gives the desired result. It therefore remains to prove the vanishing of the second part. So we have to prove the vanishing of

$$m_k w_{\dot{\alpha}}{}^{a}(\tau_i\tau_k\tau_j - \tau_j\tau_k\tau_i)^{\dot{\alpha}}{}_{\dot{\beta}}\bar{w}^{\dot{\beta}}{}_{b} \tag{4.2.38}$$

this is equal to

$$\begin{aligned}
&m_k w_{\dot{\alpha}}{}^{a}(i\tau_a\tau_i\epsilon_{jka} - i\tau_a\tau_j\epsilon_{kia} - i\tau_a\tau_k\epsilon_{jia})^{\dot{\alpha}}{}_{\dot{\beta}}\bar{w}^{\dot{\beta}}{}_{b}\\
&= m_k w_{\dot{\alpha}}{}^{a}(i\tau_a\tau_k\epsilon_{jia})^{\dot{\alpha}}{}_{\dot{\beta}}\bar{w}^{\dot{\beta}}_{b}\\
&= w_{\dot{\alpha}}{}^{a}[i\tau_a(\bar{w}w - 2\rho^2\mathbf{1})\epsilon_{jia}]^{\dot{\alpha}}{}_{\dot{\beta}}\bar{w}^{\dot{\beta}}{}_{b}
\end{aligned} \tag{4.2.39}$$

but both terms vanish because of the ADHM constraint 4.2.34. q.e.d.

And therefore we can write the gauge potential in the following form (using the normalisation $\mathrm{tr}(T^AT^B) = \frac{1}{2}\delta^{AB}$)

$$\begin{aligned}
A^I_\mu(x) &= 4\rho^2\mathrm{tr}(T^I\mathfrak{t}_c)\bar{\eta}^c_{\mu\nu}\frac{(x-x_0)^\nu}{(x-x_0)^4}\\
&= 2\rho^2\bar{\eta}^I_{\mu\nu}\frac{(x-x_0)\nu}{(x-x_0)^4}
\end{aligned} \tag{4.2.40}$$

But this is exactly the first term in the large distance ($|x-x_0| \gg \rho$) of the BPST solution in singular gauge presented in section 4.1:

$$\begin{aligned}
A^a_\mu(x;x_0;\rho) &= 2\frac{\rho^2\bar{\eta}^a_{\mu\nu}(x-x_0)^\nu}{(x-x_0)^2[(x-x_0)^2+\rho^2]}\\
&= 2\rho^2\bar{\eta}^c_{\mu\nu}\frac{(x-x_0)\nu}{(x-x_0)^4}(1 - \frac{\rho^2}{(x-x_0)^2} + \dots)
\end{aligned} \tag{4.2.41}$$

Also the higher terms in this expansion can be derived from scattering amplitudes as shown in [30].

4.2.3 E2-instantons in type IIA string theory

In our first example we did not care about the internal space when considering the properties specific for instantons. Now we want to look at instantons which are 3-dimensional cycles and therefore extended objects in the internal space.

Let us, for concreteness consider type IIA string theory compactified on a Calabi-Yau manifold. To reduce the amount of supersymmetry from $\mathcal{N} = 2$ to $\mathcal{N} = 1$ we perform an orientifold projection $\Omega\bar{\sigma}(-1)^{F_L}$ as given in chapter 2.3, where Ω denotes the world sheet parity operator, $\bar{\sigma}$ is an antiholomorphic involution and F_L denotes the number operator of left moving fermions. We therefore get $O6$-planes wrapping a (special Lagrangian) 3-cycle in the internal manifold as a fixed point set. As we saw in the last chapter, the Green-Schwarz-mechanism was used to cancel the RR-tadpoles by Chern-Simons-couplings of the RR-form C_3. If we now look for internal cycles which intersect the above objects in a point in general position and also couple to the axionic 3-forms we are guided to consider Euclidean branes with 3 dimensions.

The Dirac-Born-Infeld and Chern-Simons-actions for Ep-Instantons are of a similar structure as for Dp-branes (2.2.26 and 2.2.29). Looking at the coupling of the RR-form C_3 one can read off the action for E2- instantons

$$S = e^{-\phi} \int_{\Gamma_3} \Omega_3 + i \int_{\Gamma_3} C_3 \tag{4.2.42}$$

where Ω_3 is the holomorphic 3-form on the internal Calabi-Yau.

4.2.4 Zero modes

Now we turn to the massless spectrum of an open string with both ends on the E2-brane or with one end on the E2-brane and one end on a matter-D6-brane. Because these are very important for the phenomenological parts later in this work we will explain their properties in more detail than in the first example.

The important difference to open strings ending on Dp-branes is again the pointlike character of E2-branes in the external space. Therefore in the external dimensions we get Dirichlet-Dirichlet (DD) boundary conditions for open strings with both ends on the E2-brane and Dirichlet-Neumann (DN) boundary conditions if the open string ends on the E2-brane and on the D6-brane.

We can observe the number and properties of the different zero modes by considering the mass formulas of the superstring with appropriate boundary conditions. As in section 3.1 we get fractional modings $\alpha_{n+\epsilon_i}$ and $\psi_{n+\kappa+\epsilon_i}$ if the cycles wrapped by the branes intersect at angles ϵ_i and $\kappa = \frac{1}{2}(0)$ for the NS (R) sector. The DN-boundary conditions give an additional shift of $\frac{1}{2}$ in the external dimensions of the E2-D6-sector which has the consequence that the zero point energy in the NS-sector (which is already greater than $-\frac{1}{2}$ for generically intersecting branes) gets shifted by $\frac{1}{2}$ and therefore there will be no zero modes from this sector. In the R-sector the zero point energy is still zero but because of the $\frac{1}{2}$-shift in the mode expansion we have a world-sheet spinor as zero mode (and is thus two-dimensional). The GSO-projection will select a specific chirality and therefore we get one real degree of freedom.

We now want to describe in more detail the different zero modes and give a physical interpretation.

Zero modes in the E2-E2-sector

Due to being pointlike in four dimensions, the instanton breaks translational symmetry in these directions and therefore we get four massless Goldstone-bosons x^{μ}. In addition, some supersymmetry will be broken, depending on the geometry of the cycle wrapped by the E2-brane: In general position it will not be invariant under the orientifold projection and therefore we have the full $\mathcal{N} = 2$ supersymmetry (Q_1,Q_2) on the Calabi-Yau as constructed in the second chapter. The orientifold projection will break half of these supersymmetries: Due to the localisation of the instanton [27], the unbroken supercharges will be Q_1^{α} and $\bar{Q}_2^{\dot{\alpha}}$ leading to four Goldstino modes. To emphasize their origin of different parts of the $\mathcal{N} = 2$-algebra they will be denoted by θ^{α} and $\bar{\tau}^{\dot{\alpha}}$.

We conclude that (if we assume the absence of additional zero modes to be discussed in the next subsection) to get a F-term contribution (we finally want to integrate over the zero modes in analogy to the previous section), we have to look for a configuration where half of the Goldstino-modes are absent, in other words the configuration should only see $\mathcal{N} = 1$ supersymmetry. This is the case on an appropriately placed spacetime-filling D-brane (and therefore the E2-brane should be placed on top of the D-brane) or on a fixed-point-cycle of the orientifold-projection. The first case is similar to the first example of instantonic branes discussed in the previous subsection. It corresponds to the stringy realization of a gauge instanton. In the second case one has to be careful about the action of the orientifold map which in the case of one E2-brane projects out the $\bar{\tau}^{\dot{\alpha}}$ modes (see [27] for the general case of a stack of E2-branes).

There are also zero modes coming from the different directions in which we can deform the cycle the instantonic brane wraps. One can show that there are one complex bosonic and four fermionic zero mode associated with one direction in which we can deform the cycle. The multiplicity of these zero modes is counted by the number of possible deformations which is given for E2-instantons by the first Betti-number $b^1(E2)$. A instanton without these zero modes is called *rigid*.

Zero modes in the E2-D6 sector

Let us now turn to the case of an open string with one end on the E2-brane and one end on an N-stack of D6-branes (which contains the external space). As already noticed, there are fermionic zero modes from the Ramond sector. The GSO projection only gives rise to chiral spinors on the two-dimensional world-sheet and we therefore get a one-dimensional Grassmannian degree of freedom. These zero modes are called *charged* zero modes because they transform with respect to the fundamental representation of the gauge group given on the stack of D6-branes. In the following table 4.3 we list the multiplicities of the different representations with respect the gauge theory on the E2 and the stack of D6-branes labelled by a (which are given by the topological intersection numbers similar to the case of intersecting D6-branes).

Zero mode	Representation	Multiplicitiy
$\lambda_a := \lambda_{E2-a}$	$(-\mathbf{1}_{E2},\Box_a)$	I^+_{E2-a}
$\bar\lambda_a := \lambda_{a-E2}$	$(\mathbf{1}_{E2},\overline{\Box}_a)$	I^-_{E2-a}
$\lambda'_a := \lambda_{E2'-a}$	$(\mathbf{1}_{E2},\Box_a)$	$I^+_{E2'-a}$
$\bar\lambda'_a := \lambda_{a-E2'}$	$(-\mathbf{1}_{E2},\overline{\Box}_a)$	$I^-_{E2'-a}$

Table 4.3: Zero mode content of an instantonic E2-brane intersecting with a stack a of D6-branes

From the table we can read off the total $U(1)$-charge of the zero modes with respect to the stack a:

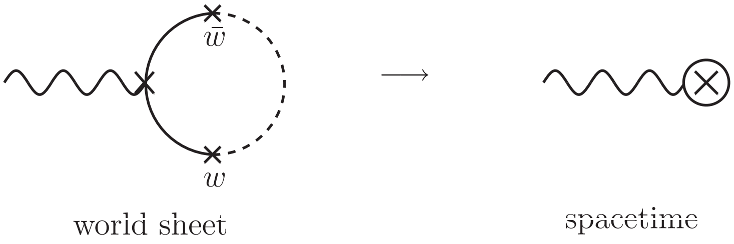
$$\begin{aligned} Q_a(E2) &= N_a(I^+_{E2-a} - I^-_{E2-a} - I^+_{E2'-a} + I^-_{E2'-a}) \\ &= N_a(I_{E2-a} - I_{E2'-a}) \end{aligned} \tag{4.2.43}$$

4.2.5 Instanton calculus in string theory

We now want to give a way to calculate string theoretic correlation functions in a background of D-instantons. To have a concrete case in mind, we go back to our first example of the D(-1)/D3 -system and take again the notation introduced there for the moduli fields. The important observation which leads us to a prescription of how to deal with insertions of moduli fields is that the moduli fields have no momentum in the four dimensional world and thus have no dynamics. We therefore can identify for example a 3-point function with two moduli fields and one gauge field with a 1-point function in spacetime. We will introduce the following notation for disc amplitudes to write this explicitly:

$$\langle V^{-1}_{\bar w} V^0_{A^I_\mu} V^{-1}_w \rangle = \langle V^0_{A^I_\mu} \rangle_{\mathcal{D}(\bar w, w)} \tag{4.2.44}$$

which also can be stated in terms of Feynman diagrams:



Where we depicted the instanton brane with dashed lines to distinguish it from the D(3)-brane, which is a solid boundary.

Now let us first consider all disc diagrams with only moduli fields inserted, i.e. in the above introduced language, spacetime-vacuum contributions (figure 4.2). In terms of correlators this reads

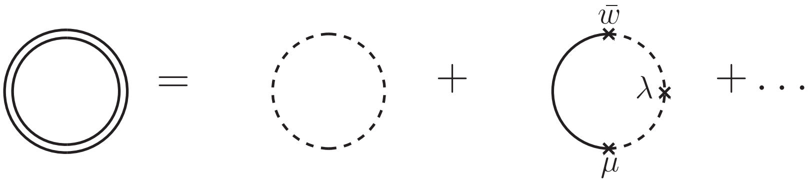


Figure 4.2: Spacetime vacuum contributions

$$\langle \mathbf{1} \rangle_{\mathcal{D}(\mathcal{M})} = \langle \mathbf{1} \rangle + \langle V_{\bar{w}} V_{\lambda} V_{\mu} \rangle + \dots \tag{4.2.45}$$

Calculating the amplitude without any insertions it was shown in [30] that it is proportional to the topological charge k. The sum of the other diagrams gives the full moduli action:

$$\langle \mathbf{1} \rangle_{\mathcal{D}(\mathcal{M})} = -\frac{8\pi^2 k}{g_{YM}^2} - S_m \tag{4.2.46}$$

The next step is to consider now also insertions of matter fields. To get the full contribution to a correlator of n matter fields ϕ_i one has to sum over discs with all possible configurations of moduli vertex operators $V_{\mathcal{M}}$ inserted:

$$\langle \prod_i V_{\phi_i} \rangle_{\mathcal{D}(\mathcal{M})} = \frac{8\pi^2}{g_{YM}^2} \sum_{conf} \int \frac{\prod_i dz_i \prod_j dy_j}{dV_{abc}} \langle \prod_k V_{\phi_k}(z_k, p_k) \prod_l V_{\mathcal{M}_l}(y_l) \rangle \tag{4.2.47}$$

Here the division by dV_{abc} means that we can fix three positions on the disc.

In addition to that, in analogy to the field theory case described earlier, we have to integrate over the space of moduli. We denote the corresponding measure by $d\mathcal{M}$.

This integration has the effect that a class of disconnected diagrams get connected in the spacetime point of view. In the integrated product

$$\int d\mathcal{M} \langle \prod_i V_{\phi_i} \rangle_{\mathcal{D}(\mathcal{M})} \langle \mathbf{1} \rangle_{\mathcal{D}(\mathcal{M})} \tag{4.2.48}$$

the integration over the moduli will connect the two parts at the location of the different moduli and therefore we get a connected diagram of the fields ϕ_i. But this works also for a product of two correlators of matter (ϕ_i) fields like

$$\int d\mathcal{M} \langle \prod_i \phi_i \rangle_{\mathcal{D}(\mathcal{M})} \langle \prod_j \phi_j \rangle_{\mathcal{D}(\mathcal{M})} \tag{4.2.49}$$

The result of the moduli-integration therefore is that we have also to consider these seemingly disconnected diagrams when computing correlation functions of matter fields in an instanton background.

In the case of equation 4.2.48 we have to sum over all kinds of products of the vacuum contribution. Taking the different symmetry factors into account ([31]) we get

$$\begin{aligned} & 1 + \langle \mathbf{1} \rangle_{\mathcal{D}(\mathcal{M})} + \frac{1}{2} \langle \mathbf{1} \rangle_{\mathcal{D}(\mathcal{M})} \langle \mathbf{1} \rangle_{\mathcal{D}(\mathcal{M})} + \dots \\ & = e^{-\langle \mathbf{1} \rangle_{\mathcal{D}(\mathcal{M})}} \end{aligned} \tag{4.2.50}$$

In addition to that as a consequence of equation 4.2.49 we have to sum over all different distributions of matter fields on disc diagrams.

To summarize, if we want to compute a (amputated) correlation function in an instanton background (at disc level) we have

$$\langle \prod_{j=1}^{n} \phi_j \rangle |_{D-inst} = \sum_{\substack{\sigma_i \subset \{1 \dots n\} \\ \dot{\bigcup} \sigma_i = \{1 \dots n\}}} \int d\mathcal{M} \langle \prod_{k_1 \in \sigma_1} V_{\phi_{k_1}} \rangle_{\mathcal{D}(\mathcal{M})} \cdots \langle \prod_{k_r \in \sigma_r} V_{\phi_{k_r}} \rangle_{\mathcal{D}(\mathcal{M})} e^{-\langle \mathbf{1} \rangle_{\mathcal{D}(\mathcal{M})}} \tag{4.2.51}$$

Finally we want to remark that the above calculation only considered scattering at disc level. To be more complete one has also to consider all different world-sheet topologies ([30]).

4.2.6 Instanton contributions to the superpotential in type IIA string theory

We now specialize to the case which is the most important one for our phenomenological investigations in the next chapters.

The goal is to find corrections to the superpotential of the four-dimensional effective supergravity action. In our phenomenological considerations we do not want to calculate the cft-correlators exactly. We just want to read off the general structure of the correction terms and estimate their order of magnitude. To do this we have to be careful to get the right supergravity normalization of the superpotential terms. A superpotential term consisting of a product of chiral superfields $\Phi_i = \phi_i + \sqrt{2}\theta\psi_i + \theta\theta F_i$ which is multiplied by the appropriate factor of the Planck mass

$$W \supset M_p^{-n} \prod_{i=1}^{n+3} \Phi_i \tag{4.2.52}$$

leads to an F-term in the supergravity action of the form

$$\frac{\partial^2 W}{\partial \Phi_i \partial \Phi_j} |_{\theta=0} e^{K/2} \psi_i \psi_j = \frac{e^{K/2} \psi_i \psi_j \prod_{k \neq i,j} \phi_k}{\sqrt{K_{ii} K_{jj} \prod_{k \neq i,j} K_{kk}}} M_p^{-n} \tag{4.2.53}$$

Here K is the Kähler potential and K_{ij} is the matter metric which we assume to be diagonal for simplicity. The square root factors come from the canonical normalisation of the matter fields. In the next step we want to estimate their strength. For this let us denote the volume of the internal Calabi-Yau by $V = \mathcal{V} l_s^6$. It can be shown ([32]) that the matter metric is inversely proportional to the real part τ_b of the Kähler modulus of one of the large cycles in the internal space and one gets the following volume dependence

$$K_{ii} \propto \tau_b^{-1} \propto \mathcal{V}^{-\frac{2}{3}} \tag{4.2.54}$$

In addition, the Kähler potential contains a term $-2 \ln \mathcal{V}$ and in 4.2.53 this leads to

$$M_P^{-n} \mathcal{V}^{\frac{n}{3}} \; \psi_i \psi_j \prod_{k \neq i,j} \phi_k \tag{4.2.55}$$

Using the relation between the string and Planck mass ([32]) $M_s = M_P / \sqrt{\mathcal{V}}$ we finally get a scale factor of $M_s^{-n} \mathcal{V}^{-\frac{n}{6}}$.

Let us now turn to the computation of the instanton contribution to the superpotential using the instanton calculus. In the section about the zero modes of E2 instantons we have seen that if the instanton is rigid and lies on top of an orientifold plane, then it has the right amount of universal zero modes to have a chance to contribute to the superpotential. In general, if the cycle of the instanton intersects the cycle on which the matter-brane-stacks are wrapped, there are in addition charged zero modes over which we have to integrate. If we focus on contributions of the order g_s^0 in the main expression 4.2.51 (which are dominant in the weak coupling region of small g_s) we see that only corrections of the form

$$\langle \Phi_{a x_1} \Phi_{x_1 x_2} \ldots \Phi_{x_n b} \rangle_{\lambda_a \bar{\lambda}_b} \tag{4.2.56}$$

contribute (because with every λ we get a factor of $\sqrt{g_s}$ and the disc carries an overall normalisation of g_s^{-1}). Thus we arrive at ([27])

$$\begin{aligned} &\langle \Phi_{a_1 x_{11}} \Phi_{x_{11} x_{12}} \ldots \Phi_{x_{1(n_1-1)} x_{1 n_1}} \Phi_{x_{1 n_1} b_1} \ldots \Phi_{a_M x_{M1}} \Phi_{x_{M1} x_{M2}} \ldots \Phi_{x_{M(n_M-1)} x_{M n_M}} \Phi_{x_{M n_M} b_M} \rangle \\ &\cong \int d^4x d^2\theta \sum_{conf} \prod_a (\prod_{i=1}^{I^+_{E2D_a}} d\lambda_a^i) (\prod_{i=1}^{I^-_{E2D_a}} d\bar{\lambda}_a^i) \\ &\quad e^{\langle \mathbf{1} \rangle_{\mathcal{D}(\mathcal{M})}} e^{\text{1-loop}} \langle \hat{\Phi}_{a_1 b_1} [\vec{x}_1] \rangle_{\lambda_{a_1} \bar{\lambda}_{b_1}} \ldots \langle \hat{\Phi}_{a_L b_L} [\vec{x}_L] \rangle_{\lambda_{a_L} \bar{\lambda}_{b_L}} \end{aligned} \tag{4.2.57}$$

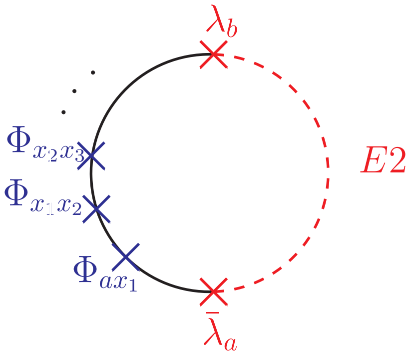
where we have introduced the notation

$$\hat{\Phi}_{a_k b_k} [\vec{x}_k] := \Phi_{a_k x_{k1}} \Phi_{x_{k1} x_{k2}} \Phi_{x_{k2} x_{k3}} \ldots \Phi_{x_{k(n_k-1)} x_{k n_k}} \Phi_{x_{k n_k} b_k} \tag{4.2.58}$$

and the correlators are pictorially given by disc diagrams as depicted in figure 4.3.

In the case of an E2-instanton, the empty disc gives us a factor of

$$-\langle \mathbf{1} \rangle = \frac{1}{g_s} \frac{\text{Vol}(E2)}{l_s^3} \tag{4.2.59}$$



$$= \quad \langle \hat{\Phi}_{ab}[\vec{x}] \rangle_{\bar{\lambda}_a \lambda_b}$$

Figure 4.3: Disc diagram with two zero modes $\bar{\lambda}_a$ and λ_b which change the boundary from the matter branes (solid lines) to the E2-instanton brane (dashed line). In addition matter fields Φ_{ij} are inserted. Between insertions Φ_{ij} and Φ_{jk} the boundary is the matter brane j.

We therefore get a supression factor of the contribution proportional to $e^{-\frac{\mathrm{Vol}(E2)}{g_s l_s^3}}$.

To sum up, if we do the integration over the zero modes we get a contribution to the superpotential (if the number of charged zero mode fields in the correlators match the number of charged zero modes in the measure, because otherwise the fermionic integration gives zero) which has the schematic form

$$\int d^4x d^2\theta \prod_{i=1}^{M} \Phi_{a_i b_i} e^{-S_{E2}} \tag{4.2.60}$$

and which has to be multiplied by the appropriate mass and scale factors like in 4.2.55.

We know from the action of the E2 brane that it also involves as imaginary part the axion described in chapter 3.3.2, and which transforms under a global $U(1)_a$-gauge symmetry (with generator Λ_a) as $\Phi \longrightarrow \Phi + Q_a(E2)$ and therefore we get the transformation law

$$e^{-S_{E2}} \longrightarrow e^{iQ_a(E2)\Lambda_a} e^{-S_{E2}} \tag{4.2.61}$$

We conclude that a instanton-generated contribution of the form 4.2.60 is invariant under the global $U(1)_a$-symmetry, if

$$\sum_i Q_a(\Phi_{a_i b_i}) = -N_a(I_{a,E2} - I_{a',E2}) \tag{4.2.62}$$

Chapter 5

Yukawa couplings/mass hierarchies in MSSM models

Now we have developed some of the most relevant tools to analyze possible phenomenological implications of D-instantons. Because of 4.2.62 it is possible to have couplings which are perturbatively forbidden because of the violation of global $U(1)$-symmetries. We use this to generate Yukawa terms such as the light quark masses. The instantonic supression factor of $e^{-\frac{\mathrm{Vol}(E2)}{g_s l_s^3}}$ can be used to get the different mass hierarchies as we observe in nature. We will demonstrate this in the next section.

But there is also a dangerous effect which often rules out MSSM models: If an instanton which is needed for example to generate a perturbatively forbidden coupling like Yukawa terms or neutrino masses, also has the right intersection (and therefore zero-mode-) structure to generate dimension 4/5-couplings which could result in baryon/lepton number violating processes which are strongly bounded by experimental observations (like proton decay), a tension arises between getting the right strength of the desired coupling and staying beyond experimental bounds. Obviously one of the strongest supressed effects is proton decay. The experimental bound is $\tau_p \geq 10^{32} y$. But also flavour violating effects are highly constrained by experiment ([44]). The contribution of instanton effects and their relation to the experimental bounds will be investigated in the next chapter.

There is also the interesting possibility to turn the way of thinking around and use experimental facts to derive bounds on the instantonic supression factors $e^{-\frac{\mathrm{Vol}(E2)}{g_s l_s^3}}$ which could lead to estimates on the characteristic length scale of the compactification manifold (if one for example assumes that the E2 brane minimizes its lengh (energy) and therefore wraps internal cycles which are of a length scale which is characteristic for the internal manifold).

5.1 Useful facts about the MSSM

But before starting to investigate the implications of D-instantons on particle physics we first want to briefly state the most relevant facts of the minimal extension of the standard model to a supersymmetric theory (called the minimal supersymmetric standard model). We only state the results and the reader is invited to have a look at the broad literature on this subject (here we used mainly the conventions of [34], [35]; an introduction to phenomenological issues is given for example in [36], [37] and also the references given there).

Particle content

To extend the standard model of particle physics into a supersymmetric version, one has to fill the particle multiplets into appropriate supermultiplets. Since supersymmetry relates fermions and bosons the idea of packing standard model fermions and gauge bosons in the same supermultiplets suggests itself immediately. However, this does not work since the bosons and fermions in a supermultiplet should transform in equal representations of the gauge group. We therefore get chiral supermultiplets for the fermions and vector supermultiplets for gauge bosons and as a consequence the superpartners arise as the other components in the respective multiplets. In addition, since there are contributions to the $U(1)^3$ and $U(1) - SU(2)^2$ anomalies coming from the Higgsinos which do not cancel, a second Higgs doublet is needed. In addition it is also needed for the Yukawa couplings of up and down-type particles in order to preserve the holomorphicity of the superpotential.

The particle content is summarized in the following table:

Superfield	Bosons	Fermions	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
Q_L	$(\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)	$\square$	$\square$	$\frac{1}{6}$
U_R	$\tilde{u}_R^*$	$u_R^\dagger$	$\bar{\square}$	$\mathbf{1}$	$-\frac{2}{3}$
D_R	$\tilde{d}_R^*$	$d_R^\dagger$	$\bar{\square}$	$\mathbf{1}$	$\frac{1}{3}$
L	$(\tilde{\nu}, \tilde{e}_L)$	(ν, e_L)	$\mathbf{1}$	$\square$	$-\frac{1}{2}$
E_R	$\tilde{e}_R^*$	$e_R^\dagger$	$\mathbf{1}$	$\mathbf{1}$	1
H_u	(H_u^+, H_u^0)	$(\tilde{H}_u^+, \tilde{H}_u^0)$	$\mathbf{1}$	$\square$	$\frac{1}{2}$
H_d	(H_d^0, H_d^-)	$(\tilde{H}_d^0, \tilde{H}_d^-)$	$\mathbf{1}$	$\square$	$-\frac{1}{2}$
G	G_μ^a	$\tilde{G}^a$	Ad	$\mathbf{1}$	0
W	$W_\mu^\pm, W_\mu^3$	$\tilde{W}^\pm, \tilde{W}^3$	$\mathbf{1}$	Ad	0
B	B_μ	$\tilde{B}$	$\mathbf{1}$	$\mathbf{1}$	0

Table 5.1: Spectrum of the MSSM

where we wrote down one generation. To indicate all three generations we often use the notation

$$Q_L^I, \quad U_R^I, \quad D_R^I, \qquad I = 1\ldots 3 \tag{5.1.1}$$

and similar for the other matter fields.

Lagrangian and couplings of the MSSM

We now give the superspace formulation of the Lagrangian of the MSSM. We will need the structure of the couplings later on, when we try to derive some phenomenology induced by D-istantons which are needed in specific intersecting brane models to generate required couplings. The strategy later will be to use the MSSM vertices and combine them with D-instanon-generated ones to get processes which may be observable and to get hints how processes could be realized at the 1-loop level in string theory.

Since they are not needed later we omit the gauge fixing/ghost parts.

Let us begin with the kinetic terms for the gauge fields

$$\mathcal{L}_g = \int d^2\theta \frac{1}{4}[2\mathrm{tr}(\mathbf{W}_w \mathbf{W}_w) + W_e W_e + 2\mathrm{tr}(\mathbf{W}_{qcd}\mathbf{W}_{qcd})] + h.c. \tag{5.1.2}$$

Where we have $\mathbf{W}_{w\alpha} = \bar{D}\bar{D}e^{-\mathbf{V}}D_\alpha e^{\mathbf{V}}$ and $\mathbf{V} = \sum_{a=1}^{3} \frac{\tau^a}{2} V^a$ for the weak interaction and similar for the strong interaction $\mathbf{W}_{qcd}$ with $\mathbf{V}_{qcd} = \sum_{b=1}^{3} \frac{\lambda^b}{2} V_{qcd}^b$ and $W_\alpha = -\frac{1}{4}\bar{D}\bar{D}D_\alpha V$ for electromagnetism. Here the D_α is the covariant derivative in superspace.

Then the kinetic terms of the matter superfields

$$\begin{aligned}
\mathcal{L}_k = &\int d^2\theta d^2\bar{\theta}\ L^\dagger \exp(g\tau^a V^a + g' Y_l V) L \\
&+ \int d^2\theta d^2\bar{\theta}\ E_R^\dagger \exp(g' Y_{e_R} V) E_R \\
&+ \int d^2\theta d^2\bar{\theta}\ Q_L^\dagger \exp(g\tau^a V^a + g' Y_q V + g_{qcd}\lambda^b V_{qcd}^b) Q_L \\
&+ \int d^2\theta d^2\bar{\theta}\ U_R^\dagger \exp(g' Y_u V - g_{qcd}\lambda^b V_{qcd}^b) U_R \\
&+ \int d^2\theta d^2\bar{\theta}\ D_R^\dagger \exp(g' Y_d V - g_{qcd}\lambda^{*b} V_{qcd}^b) D_R \\
&+ \int d^2\theta d^2\bar{\theta}\ H_u^\dagger \exp(g\tau^a V^a + g' Y_{H_u} V) H_u \\
&+ \int d^2\theta d^2\bar{\theta}\ H_d^\dagger \exp(g\tau^a V^a + g' Y_{H_d} V) H_d
\end{aligned} \tag{5.1.3}$$

Now there are the Yukawa terms (we omit the exact normalisation)

$$\mathcal{L}_Y = \int d^2\theta \; H_u L E_r + H_u Q_L U_R + H_d Q_L D_R - \mu H_u H_d + h.c. \qquad (5.1.4)$$

In addition there are soft breaking terms, which break supersymmetry but do not introduce new quadratic divergences: We will use later only $-m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - (bH_u H_d + h.c.)$. Finally we also can add the following terms which usually induce dangerous processes like proton decay which is too fast and therefore one introduces the R-pariy ([35]) symmetry which forbids them:

$$\mathcal{L}_R = \alpha_{IJK} Q_L^I L^J D_R^K + \beta_{IJK} L^I L^J E_R^K + \gamma_I L^I H_u + \delta_{IJK} D_R^I D_R^J U_R^K \qquad (5.1.5)$$

The component Lagrangian is rather long and can for example be found in [38] and [39].

5.2 Yukawa couplings/mass hierarchies from D-instantons

Let us first discuss the constraints which one has from theoretical considerations (often called top-down-constraints). To illustrate the ideas we also discuss three different examples with three, four and five D-brane stacks to realize the gauge symmetries and matter content of the MSSM. In every model, the first two stacks should represent the non-Abelian symmetries and the others are different $U(1)$-factors, i.e. the gauge symmetry is

$$U(3)_a \times U(2)_b \times U(1)_c \times U(1)_d \times \dots \qquad (5.2.6)$$

As we know from chapter 3.3 there are no non-Abelian anomalies due to the tadpole cancellation condition and the mixed, Abelian and gravitational anomalies are cancelled by the Green-Schwarz-mechanism. The tadpole condition in the case of IIA with spacetime filling branes wrapping internal 3-cycles Π_a reads

$$\sum_n N_n(\Pi_n + \Pi_n') - 4\Pi_{O6} = 0 \qquad (5.2.7)$$

If we multiply this with the homology class of a specific cycle Π_a this leads to a condition on the muliplicities of the matter representations (remember table 3.1)

$$\begin{aligned} &\sum_n N_n(\Pi_a \circ \Pi_n + \Pi_a \circ \Pi_n') - 4\Pi_a \circ \Pi_{O6} = 0 \\ \Rightarrow &\sum_{n \neq a} N_n(\Pi_n \circ \Pi_a' - \Pi_n \circ \Pi_a) + N_a \Pi_a \circ \Pi_a' - 4\Pi_a \circ \Pi_{O6} = 0 \end{aligned} \qquad (5.2.8)$$

which can be rewritten appropriate to table 3.1

$$\sum_{n\neq a} N_a(\Pi_n\circ\Pi'_a-\Pi_n\circ\Pi_a)+(\frac{N_a-4}{2})(\Pi_a\circ\Pi'_a+\Pi_a\circ\Pi_{O6})+(\frac{N_a+4}{2})(\Pi_a\circ\Pi'_a-\Pi_a\circ\Pi_{O6})=0 \tag{5.2.9}$$

and therefore we get the condition

$$\#(\mathbf{a})-\#(\bar{\mathbf{a}})+(N_a-4)\#(\begin{smallmatrix}\square\\\square\end{smallmatrix}_a)+(N_a+4)\#(\square\square_a)=0 \tag{5.2.10}$$

where $\#(\dots)$ denotes the number of fields transforming in the representation given in brackets.

The Green-Schwarz mechanism implies that the gauge bosons of the $U(1)$'s generically acquire a mass (but remain gauge symmetries due to the Stückelberg mechanism). To have the standard model massless hypercharge we require at least one massless $U(1)$ to exist, whose generator is a linear combination of the different $U(1)$-generators:

$$U(1)_Y=\sum_x q_x U(1)_x \tag{5.2.11}$$

As we have seen in section 3.3 this combination remains massless, if the condition 3.3.24 holds. If we multiply it again with the homology class of a specific cycle and perform the same calculation as above, we get a second condition

$$-q_a N_a(\#(\square\square_a)+\#(\begin{smallmatrix}\square\\\square\end{smallmatrix}_a))+\sum_{x\neq a} q_x N_x \#(\square_a,\bar{\square}_x)-\sum_{x\neq a} q_x N_x \#(\square_a,\square_x)=0 \tag{5.2.12}$$

In addition to these constraints one has to adjust to various experimental observations (often referred to as bottom-up-constraints). First of all we get stacks from which the MSSM matter fields arise. In addition there are usually more stacks to ensure for example tadpole cancellation (these latter correspond to a hidden sector). One requires that there are no more than the MSSM fields in the massless open string spectrum coming from the matter brane stacks and all MSSM fields should only be charged under the gauge symmetry of these stacks, i.e. we do not allow intersections between the matter stacks and the additional ones. In the concrete realization of the MSSM all the Yukawa terms in the superpotential (5.1.4) should be generated. If they are forbidden because they violate global $U(1)$-symmetries, an instanton with appropriate intersection structure is required. In addition, depending on how neutrino masses are realized (e.g. seesaw mechanism) a right handed neutrino mass term $N_R N_R$ is also needed. The R-parity violating couplings of equation(5.1.5) should be forbidden (and therefore all instantons which also induce these terms) or at least highly supressed.

More on the various phenomenological constraints can be found in [41] - [43] where also the realization in terms of D-instantons is discussed and a systematic survey for realistic models of the MSSM is performed.

5.3 D-instanton induced MSSM phenomenology

In the following section we want to discuss three examples which are one of the most promising ([43]) models of the MSSM with intersecting D-branes. The standard way to get the Yukawa couplings is to consider maps from world-sheet discs (with the matter fields contributing to the Yukawa coupling inserted) to the target space such that the different boundaries connect the intersections which give the matter fields of the Yukawa coupling. The strength of the coupling depends on the volume of the surface spanned by these intersections (for more details, see for example [40]) and therefore different coupling values can be explained geometrically.

In many cases, D-instantons are used to generate desired phenomenological properties of the respective intersecting brane models. In the rest of this work we will concentrate on these effects and remark only as an aside on possible additional effects of world-sheet instantons. Furthermore the underlying internal geometry will not be fixed. We only assume the existence of geometries which have the needed cycles and intersection structure.

5.3.1 3-stack-models

In this class of models we have the gauge symmetry

$$U(3)_a \times U(2)_b \times U(1)_c \tag{5.3.13}$$

Two different ways of realizing the massless hypercharge were found (given in [41]) which are consistent with the top down constrains and correctly give the MSSM hypercharges:

$$\begin{aligned} U(1)_Y &= \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c \quad \text{and} \\ U(1)_Y &= -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b \end{aligned} \tag{5.3.14}$$

For every combination, there are various possibilities to realize the MSSM matter fields. To illustrate the way of investigating such models we choose one specific realization with the first hypercharge and matter content given in table 5.2 ([41]).

From the $U(1)$-charges we can check which Yukawa couplings are allowed perturbatively (we indicate the charges as subscript and the family indices $I, J = 1, 2, 3$ as superscript):

$$\begin{aligned} &Q^I_{L(1,-1,0)} H_{u(0,1,1)} U^J_{R(-1,0,-1)} \quad && L^I_{(0,1,-1)} H_{d(0,-1,-1)} E^J_{R(0,0,2)} \\ &L^I_{(0,1,-1)} H_{u(0,1,1)} N^J_{R(0,-2,0)} && H_{u(0,1,1)} H_{d(0,-1,-1)} \end{aligned} \tag{5.3.15}$$

Clearly there is one forbidden Yukawa coupling:

$$Q^I_{L(1,-1,0)} H_{d(0,-1,-1)} D^J_{R(2,0,0)} \tag{5.3.16}$$

Sector	Matter	Representation	Multiplicity	Hypercharge
ab	Q_L	$(a, \bar{b})$	3	$\frac{1}{6}$
ac'	U_R	$(\bar{a}, \bar{c})$	3	$-\frac{2}{3}$
aa'	D_R	$\begin{smallmatrix}\square\\\square\end{smallmatrix}_a$	3	$\frac{1}{3}$
bc	L	$(b, \bar{c})$	3	$-\frac{1}{2}$
bc'	H_u	(b, c)	1	$\frac{1}{2}$
bc'	H_d	$(\bar{b}, \bar{c})$	1	$-\frac{1}{2}$
bb'	N_R	$\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}_b$	3	0
cc'	E_R	$\square\square_c$	3	1

Table 5.2: Spectrum of an MSSM model with hypercharge $\frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c$

where we can read off the following charges for the generating instanton

$$Q_a(E2) = -3 \quad Q_b(E2) = 2 \quad Q_c(E2) = 1 \tag{5.3.17}$$

Now from the condition 4.2.62 we can determine a possible intersection structure of the instanton such that it can generate the coupling 5.3.16:

$$I_{E2a} = -1 \quad I_{E2b} = 1 \quad I_{E2c} = 1 \tag{5.3.18}$$

where we assume (to avoid additional deformation zero modes and to get the right universal zero mode structure) that the instantonic cycle is rigid and lies on top of an orientifold plane. We therefore get zero modes which have the following transformation behaviour:

$$\begin{aligned} \bar{\lambda}_a\ \bar{\lambda}_a\ \bar{\lambda}_a :&\quad (\mathbf{1}_{E2}, \bar{\square}_a) \\ \lambda_b\ \lambda_b :&\quad (-\mathbf{1}_{E2}, \square_b) \\ \lambda_c :&\quad (-\mathbf{1}_{E2}, \square_c) \end{aligned} \tag{5.3.19}$$

The path integral contribution leading in g_s giving the Yukawa term is

$$\int d^4x d^2\theta d^3\bar{\lambda}_a d^2\lambda_b d\lambda_c e^{-S_{cl}} e^{\text{1-loop}} \langle \bar{\lambda}_a Q_L^I \lambda_b \rangle \langle \lambda_b H_d \lambda_c \rangle \langle \bar{\lambda}_a D_R^J \bar{\lambda}_a \rangle \tag{5.3.20}$$

which leads to a contribution of the form (where we absorb all additional factors in the constant Y^{IJ})

$$\int d^4x d^2\theta \;\; e^{-S_{cl}} Y^{IJ} Q_L^I H_d D_R^J \tag{5.3.21}$$

Here we can already see the effect on the mass hierarchies between the three families: The non-perturbatively realized couplings are in general supressed by the exponential factor. Depending on the factor Y this can lead to realistic hierarchies in the MSSM model. We investigate these issues more detailed in the cases of four- and five-stack-quivers (those will be the most relevant for further investigations in later chapters) in the next sections.

5.3.2 4-stack model

A very economic and beautiful way of modelling the MSSM is to take an additional $U(1)$-brane, which leads to the gauge group

$$U(3)_a \times U(2)_b \times U(1)_c \times U(1)_d \tag{5.3.22}$$

It was shown in [42] that there are the following possible massless hypercharges which give the correct hypercharge values of the MSSM

$$\begin{aligned}
U(1)_Y =& -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b \\
U(1)_Y =& -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b - \frac{1}{2}U(1)_d \\
U(1)_Y =& -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b + U(1)_d \\
U(1)_Y =& \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c \\
U(1)_Y =& \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c + \frac{1}{2}U(1)_d \\
U(1)_Y =& \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c - \frac{3}{2}U(1)_d
\end{aligned} \tag{5.3.23}$$

As a realistic example we pick out and discuss an example with the hypercharge assignment (also called the Madrid-embedding, see [42])

$$U(1)_Y = \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c + \frac{1}{2}U(1)_d \tag{5.3.24}$$

One possible realization (which turns out to be the most realistic four-stack at the moment, for a systematic analysis with respect to the bottom up constraints, see [43]) is given by

Sector	Matter fields	Representation	Multiplicity	Hypercharge
ab	Q_L	$(a,\bar{b})$	3	$\frac{1}{6}$
ac'	U_R^1	$(\bar{a},\bar{c})$	1	$-\frac{2}{3}$
ad'	U_R^{23}	$(\bar{a},\bar{d})$	2	$-\frac{2}{3}$
aa'	D_R	$\substack{\square\\\square}_a$	3	$\frac{1}{3}$
bc	L	$(b,\bar{c})$	3	$-\frac{1}{2}$
bc'	$H_u + H_d$	$(b,c) + (\bar{b},\bar{c})$	1	$\frac{1}{2} \quad -\frac{1}{2}$
bb'	N_R	$\overline{\substack{\square\\\square}}_b$	3	0
cd'	E_R	(c,d)	3	1

Table 5.3: Spectrum for the 4-stack model

We get the following perturbatively allowed Yukawa couplings (we write the $U(1)$-charges as subscript):

$$\begin{aligned} & Q^I_{L\ (1,-1,0,0)} H_{u\ (0,1,1,0)} U^1_{R\ (-1,0,-1,0)} \\ & L^I_{(0,1,-1,0)} H_{u\ (0,1,1,0)} N^J_{R\ (0,-2,0,0)} \\ & H_{u\ (0,1,1,0)} H_{d\ (0,-1,-1,0)} \end{aligned} \tag{5.3.25}$$

To complete the set of quark Yukawa couplings we have to add the following

$$Q^I_{L\ (1,-1,0,0)} H_{u\ (0,1,1,0)} U^{23}_{R\ (-1,0,0,-1)} \tag{5.3.26}$$

which has the $U(1)$-charges

$$Q_a = 0 \quad Q_b = 0 \quad Q_c = 1 \quad Q_d = -1 \tag{5.3.27}$$

and therefore has to be generated by D-instantons. From equation 4.2.62 we get the following intersection structure:

$$I_{E2,a} = 0 \quad I_{E2,b} = 0 \quad I_{E2,c} = -1 \quad I_{E2,d} = 1 \tag{5.3.28}$$

Thus we get as Yukawa matrix for the up-type quarks

$$M^{IJ}_u = \langle H_u \rangle \begin{pmatrix} g^u_{11} & g^u_{12} e^{-S^{cl}_{E2}} & g^u_{13} e^{-S^{cl}_{E2}} \\ g^u_{21} & g^u_{22} e^{-S^{cl}_{E2}} & g^u_{23} e^{-S^{cl}_{E2}} \\ g^u_{31} & g^u_{32} e^{-S^{cl}_{E2}} & g^u_{33} e^{-S^{cl}_{E2}} \end{pmatrix} \tag{5.3.29}$$

where the first column g_{I1} come from the perturbatively realized coupling. Because there is no supression factor, we associate the mass coming from these couplings to the top-quark.

The structure of the mass matrix results in masses for the three families related in the following way:

$$m_t : m_c : m_u \approx 1 : e^{-S^{cl}_{E2}} : e^{-S^{cl}_{E2}}. \tag{5.3.30}$$

Thus we get a hierarchy between the top quark and the two light families, which is required to get a realistic model. The mass difference between the two light families could be realized by different world-sheet instanton contributions (which we absorbed in the g_{IJ}).

The above instanton gives rise to an additional problem. The zero modes also give the following contribution to the path integral:

$$\int d^4x d^2\theta d\bar{\lambda}_c d\lambda_d \langle \bar{\lambda}_c E_R U_R D_R U_R \lambda_d \rangle \tag{5.3.31}$$

which gives rise to a baryon-number violating coupling which also induces effects like proton decay, which are highly supressed. We therefore see an interesting interplay between a relatively low supression factor for the instanton in order to get the right Yukawa coupling and the requirement of not violating experimental bounds on proton decay, which are very strong.

Another interesting phenomenon arises in the down-type sector. Here, the complete set of Yukawa couplings is realized non-perturbatively:

$$Q^I_{L\ (1,-1,0,0)} H_{d\ (0,-1,-1,0)} D^I_{R\ (2,0,0,0)} \tag{5.3.32}$$

We get as $U(1)$-charges:

$$Q_a = 3 \quad Q_b = -2, \quad Q_c = -1, \quad Q_d = 0. \tag{5.3.33}$$

and therefore the intersection numbers of the generating instanton are

$$I_{E2,a} = -1 \quad I_{E2,b} = 1 \quad I_{E2,c} = 1 \quad I_{E2,d} = 0 \tag{5.3.34}$$

The path integral gives the contribution

$$\int d^3\bar{\lambda}_a d^2\lambda_b d\lambda_c Y^I_{Q_L} \langle \bar{\lambda}_a Q^I_L \lambda_b \rangle Y_{H_d} \langle \lambda_b H_d \lambda_c \rangle Y^J_{D_R} \langle \bar{\lambda}_a D^J_R \bar{\lambda}_a \rangle \tag{5.3.35}$$

which leads to the factorizable Yukawa matrix

$$Y^{IJ} = Y_{H_d} Y^I_{q_L} Y^J_{d_R} \tag{5.3.36}$$

Therefore only one eigenvalue is nonzero and we need 3 different instantons of the type 5.3.34 to get 3 massive families. Using cycles with 3 different volumes we get for the ratio of masses

$$m_b : m_s : m_d \approx e^{-S^{cl}_{E2,1}} : e^{-S^{cl}_{E2,2}} : e^{-S^{cl}_{E2,3}}. \tag{5.3.37}$$

Let us now turn to the lepton-Yukawa-couplings. From the $U(1)$-charges, we see that they are forbidden perturbatively:

$$L^I_{(0,1,-1,0)} H_{d\ (0,-1,-1,0)} E^J_{R\ (0,0,1,1)} \tag{5.3.38}$$

and therefore we need an instanton of the following type

$$I_{E2,a} = 0 \quad I_{E2,b} = 0 \quad I_{E2,c} = 1 \quad I_{E2,d} = -1 \tag{5.3.39}$$

The path integral gives the contribution

$$\int d\lambda_c d\bar{\lambda}_d e^{-S^{cl}_{E2}} Y^{IJ} \langle \lambda_c H_d L^I E^J_R \bar{\lambda}_d \rangle \tag{5.3.40}$$

and thus there is no factorizable Yukawa structure. All lepton masses are supressed by the same order, which is not what we observe.

There is another possibility for an instanton to have the right charges:

$$I_{E2,a}=0 \quad I_{E2,b}=0 \quad I_{E2,c}=1 \quad I_{E2,d}=-1 \quad I^{\mathcal{N}=2}_{E2,c}=1 \tag{5.3.41}$$

where the last entry is a so-called vectorlike-intersection, which gives the zero modes $\lambda_c, \bar{\lambda}_c$ and has zero net charge ([41]). The path integral now contains the following contributions:

$$\begin{aligned}&\int d^4x d^2\theta d\lambda_c^1 d\lambda_c^2 d\bar{\lambda}_c d\bar{\lambda}_d e^{-S^{cl}_{E2}} Y^I \langle \bar{\lambda}_c E_R^I \bar{\lambda}_d\rangle (Y^J_{12}\langle \lambda_c^1 H_d L^J \lambda_c^2\rangle + Y^J_{21}\langle \lambda_c^2 H_d L^J \lambda_c^1\rangle)\\ =&\int d^4x d^2\theta e^{-S^{cl}_{E2}} Y^I (Y^J_{12}-Y^J_{21}) E_R^I H_d L^J \end{aligned}\tag{5.3.42}$$

where the minus sign comes from the exchange of the fermionic variables λ_c^1, λ_c^2.

Thus we now get a factorizable Yukawa matrix

$$Y^{IJ}=Y^I(Y^J_{12}-Y^J_{21}) \tag{5.3.43}$$

which, like in the down quark sector, allows us to use 3 different instantons to get realistic mass hierarchies:

$$m_\tau : m_\mu : m_e \approx e^{-S^{cl}_{E2,1}} : e^{-S^{cl}_{E2,2}} : e^{-S^{cl}_{E2,3}} \tag{5.3.44}$$

5.3.3 5-stack model

The addition of one $U(1)$ brane to the 4-stack model discussed in the last subsection enlarges the number of possibilities to realize the MSSM matter fields and therefore it is more likely to find a model which is more realistic. In [43] one of the closest models to the MSSM which can account for lots of the various phenomenological constraints was proposed. It uses the “extended Madrid” hypercharge embedding:

$$U(1)_Y=\frac{1}{6}U(1)_a+\frac{1}{2}U(1)_c+\frac{1}{2}U(1)_d+\frac{1}{2}U(1)_e \tag{5.3.45}$$

One choice of matter content is given in table 5.4.

The up-quark-Yukawa-couplings are the following

$$\begin{aligned}&Q^1_{L\ (1,-1,0,0,0)} H_{u\ (0,1,1,0,0)} U^{12}_{R\ (-1,0,-1,0,0)}\\ &Q^1_{L\ (1,-1,0,0,0)} H_{u\ (0,1,1,0,0)} U^{3}_{R\ (-1,0,0,-1,0)}\\ &Q^{23}_{L\ (1,1,0,0,0)} H_{u\ (0,1,1,0,0)} U^{12}_{R\ (-1,0,-1,0,0)}\\ &Q^{23}_{L\ (1,1,0,0,0)} H_{u\ (0,1,1,0,0)} U^{3}_{R\ (-1,0,0,-1,0)}\end{aligned}\tag{5.3.46}$$

Only the first one is realized perturbatively, the others are generated by appropriate instantons. Proceeding like in the last subsection we get the Yukawa-matrix

$$\langle H_u\rangle \begin{pmatrix} g^u_{11} & g^u_{12} & g^u_{13}e^{-S^{cl}_{E2,1}}\\ g^u_{21}e^{-S^{cl}_{E2,2}} & g^u_{22}e^{-S^{cl}_{E2,2}} & g^u_{23}e^{-S^{cl}_{E2,3}}\\ g^u_{31}e^{-S^{cl}_{E2,2}} & g^u_{32}e^{-S^{cl}_{E2,2}} & g^u_{33}e^{-S^{cl}_{E2,3}}\end{pmatrix} \tag{5.3.47}$$

Sector	Matter	Transformation	Multiplicity	Hypercharge
ab	Q_L^1	$(a,\bar{b})$	1	$\frac{1}{6}$
ab'	Q_L^{23}	(a,b)	2	$\frac{1}{6}$
ac'	U_R^{12}	$(\bar{a},\bar{c})$	2	$-\frac{2}{3}$
ad'	U_R^3	$(\bar{a},\bar{d})$	1	$-\frac{2}{3}$
aa'	D_R	$\begin{smallmatrix}\square\\\square\end{smallmatrix}_a$	3	$\frac{1}{3}$
bc'	H_u	(b,c)	1	$\frac{1}{2}$
bd'	L	$(\bar{b},\bar{d})$	3	$-\frac{1}{2}$
be'	H_d	$(\bar{b},\bar{e})$	1	$\frac{1}{2}$
ce'	E_R^{12}	(c,e)	2	1
ce	N_R^1	$(\bar{c},e)$	1	0
dd'	E_R^3	$\square\!\square_d$	1	1
de	N_R^{23}	$(\bar{d},e)$	2	0

Table 5.4: Spectrum for the 5-stack model

and similar for the down quark and lepton mass matrices. Again we get a hierarchical structure for the quark masses which we want for phenomenological reasons.

For completeness we finally list the charges of all Yukawa couplings in this model:

$$\begin{array}{lr}
Q_L^1 H_u U_R^{12} & 0\;0\;0\;0\;0 \\
Q_L^1 H_u U_R^3 & 0\;0\;1\;-1\;0 \\
Q_L^{23} H_u U_R^{12} & 0\;2\;0\;0\;0 \\
Q_L^{23} H_u U_R^3 & 0\;2\;1\;-1\;0 \\
Q_L^1 H_d D_R & 3\;-2\;0\;0\;-1 \\
Q_L^{23} H_d D_R & 3\;0\;0\;0\;1 \\
L H_d E_R^{12} & 0\;-2\;1\;-1\;0 \\
L H_d E_R^3 & 0\;-2\;0\;1\;-1 \\
L H_u N_R^1 & 0\;0\;0\;-1\;1 \\
L H_u N_R^{2,3} & 0\;0\;1\;-2\;1
\end{array} \tag{5.3.48}$$

To conclude, we observe that there is one heavy quark family (whose Yukawa coupling is realized perturbatively) and all other couplings have to be realized by D-instantons. Like in the 4-stack model, the appropriate choice of intersection structures can lead to realistic mass hierarchies.

Chapter 6

Flavour violating effects in MSSM orientifold models

6.1 Flavour violation in the quark sector

Let us consider the standard model Yukawa couplings which have the general form

$$\mathcal{L}_Y = -Y_u^{IJ}\epsilon^{ab}\bar{q}_{La}^I\phi_b u_R^J - Y_d^{IJ}\bar{q}_L^I \cdot \phi d_R^J + h.c. \tag{6.1.1}$$

To change to the mass eigenstate basis where the Y are diagonal, one does a rotation with unitary matrices U and A

$$Y_{u,d} = U_{u,d}\Lambda_{u,d}A_{u,d}^{\dagger} \tag{6.1.2}$$

where $\Lambda_{u,d}$ are diagonal matrices. To compensate this rotation, we also change the quark fields into the eigenstate basis:

$$\begin{aligned} u_L^I &\to U_u^{IJ}u_L^J & \qquad u_R^I &\to A_u^{IJ}u_R^J \\ d_L^I &\to U_d^{IJ}d_L^J & \qquad d_R^I &\to A_d^{IJ}d_R^J \end{aligned} \tag{6.1.3}$$

The rotation mixes flavours in the charged current weak interactions e.g. the one coupling to W^+:

$$J^{\mu\ +} \longrightarrow \frac{1}{\sqrt{2}}\bar{u}_L^I\gamma^\mu(U_u^\dagger U_d)^{IJ}d_L^J \tag{6.1.4}$$

where we define the Cabibbo-Kobayashi-Maskawa (CKM)-matrix in the usual way:

$$(U_u^\dagger U_d)^{IJ}d_L^J := Vd_L^J := d_L^{J}{}' \tag{6.1.5}$$

Thus in general there will be intergenerational mixings in charged current weak interactions. However, in the neutral current

$$\frac{g}{2\cos\theta_w}(\bar{u}_I\gamma^\mu(P_L - \frac{4}{3}\sin^2\theta_w)u_I - \bar{d}_I'\gamma^\mu(P_L - \frac{2}{3}\sin^2\theta_w)d_I' \tag{6.1.6}$$

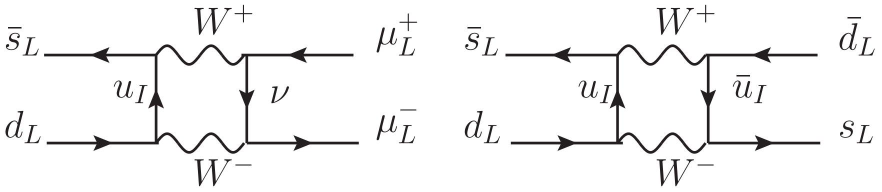


Figure 6.1: Standard model box diagrams for neutral Kaon decay and mixing

($P_L = \frac{1}{2}(1 - \gamma_5)$ and θ_w is the Weinberg angle) one can show that we can replace d'_I by d_I and therefore there are no tree level flavour changing neutral currents (fcnc's). This mechanism was first realized by Glashow Ilionopoulos and Maiani (GIM) who used it to predict the charm quark. The leading order fcnc processes can only show up at the one-loop level, for example in the mixing of the K^0-meson or its decay (figure 6.1).

In supersymmetric extensions of the standard model (focussing on the MSSM) there are new contributions coming from quarks and gluinos in the loop. Since we have observed a very strong supression of these effects (for a recent review of the experimental status, see [44]), we require a very strong supression of these effects and therefore we get bounds on the parameters of the theory, for example the squark mixing angles.

6.1.1 Flavour violation in 5-stack quivers

We now begin to investigate possible flavour violating effects in MSSM models. Firstly, there are perturbatively allowed contributions to quark flavour mixing, as pointed out in [49]. They are of the four fermion- type and therefore supressed by the square of the string mass scale M_s:

$$\frac{1}{M_s^2} \langle \bar{\psi} \psi \bar{\psi} \psi \rangle \tag{6.1.7}$$

If in an MSSM model at least two families are realized by the same representation, these four-fermion operators can contribute to flavour mixing. The strongest bounds come from neutral kaon mixing and were calculated in [49]. They give a lower bound on the string scale of $M_s > 10^{3-4}$ TeV.

But in addition there are also instanton contributions, which we investigate in detail in the following.

In intersecting brane models of the MSSM we observed in chapter 5 that not every phenomenologically required Yukawa coupling can be realized perturbatively, because of the violation of global $U(1)$-symmetries. Therefore we need E2-instantons (in the case of type IIA theory) to generate the missing couplings.

Because we have to sum over all allowed insertions of moduli and matter fields in the path integral, we also get additional phenomenologically interesting contributions from the "Yukawa-instantons". Furthermore the coupling strength of the instanton is set by the strength of the Yukawa couplings and therefore the new effects give us bounds on the other parameters involved, like the string mass scale. In the following we want to search systematically for phenomenologically interesting effects generated by Yukawa-instantons. Because we know the supression scale of the instanton, we can infer from experimental bounds on highly supressed processes some lower bounds on the fundamental parameters in the model, like the string scale.

As a concrete model, we use the following 5-stack MSSM-quiver, first discussed in the work [45]. It has the hypercharge

$$U(1)_Y = \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c + \frac{1}{2}U(1)_d + \frac{1}{2}U(1)_e \tag{6.1.8}$$

Table 6.1: Spectrum of the 5-stack quiver

Sector	Matter Fields	Representation	Multiplicity	Hypercharge
ab	Q_L^1	$(a, \bar{b})$	1	$\frac{1}{6}$
ab'	$Q_L^{2,3}$	(a, b)	2	$\frac{1}{6}$
ac	D_R	$(\bar{a}, c)$	3	$\frac{1}{3}$
ac'	$U_R^{1,2}$	$(\bar{a}, \bar{c})$	2	$-\frac{2}{3}$
ae'	U_R^3	$(\bar{a}, \bar{e})$	1	$-\frac{2}{3}$
bc	H_u	$(\bar{b}, c)$	1	$\frac{1}{2}$
bc'	H_d	$(\bar{b}, \bar{c})$	1	$-\frac{1}{2}$
bd'	$L^{1,2}$	$(\bar{b}, \bar{d})$	2	$-\frac{1}{2}$
be	L^3	$(b, \bar{e})$	1	$-\frac{1}{2}$
cd'	E_R^1	(c, d)	1	1
ce'	$E_R^{2,3}$	(c, e)	2	1

The quark and lepton Yukawa couplings have the following $U(1)$-charges :

$$\begin{array}{lr}
Q_L^1 U_R^{12} H_u & 0, -2, 0, 0, 0 \\
Q_L^1 U_R^3 H_u & 0, -2, 1, 0, -1 \\
Q_L^{2,3} U_R^{1,2} H_u & 0, 0, 0, 0, 0 \\
Q_L^{2,3} U_R^3 H_u & 0, 0, 1, 0, -1 \\
Q_L^1 D_R H_d & 0, -2, 0, 0, 0, \\
Q_L^{2,3} D_R H_d & 0, 0, 0, 0, 0 \\
L^{1,2} H_d E_R^1 & 0, -2, 0, 0, 0 \\
L^{1,2} H_d E_R^{2,3} & 0, -2, , 0, -1, 1 \\
L^3 H_d E_R^1 & 0, 0, 0, 1, -1 \\
L^3 H_d E_R^{2,3} & 0, 0, 0, 0, 0
\end{array} \tag{6.1.9}$$

For these instantons we performed a systematic analysis of the phenomenology that contributes to the path integral (up to operators of mass dimension smaller or equal to 5). More precisely, we listed all possible holomorphic chain products of matter fields which are possible insertions in disc amplitudes together with the charged zero modes coming from the intersection of the Yukawa-instantons with the matter branes. In some exceptional cases which seemed to be very interesting, also non-holomorphic products are listed. The rules of a chain product to give an allowed contribution were described in chapter 4.2. The results of the analysis are given in the tables in appendix 3. There are various interesting operators which we summarize and comment in the following:

1. $\boxed{0,-2,0,0,0}$

   If the charges are realized with two zero modes λ_b, λ_b we get the following interesting dimension 5 operators:

$$\begin{array}{ll} Q^1_L D_R U^{12}_R Q^1_L & Q^1_L U^{12}_R D_R Q^1_L \\ Q^1_L U^{12}_R E^1_R L^{12} & H_d E^{23}_R U^3_R Q^1_L \\ L^{12} E^{23}_R U^{12}_R Q^1_L & \end{array} \tag{6.1.10}$$

   The second way to realize this charge structure is with zero modes $\lambda_b, \lambda_b, \lambda_c\bar{\lambda}_c$. If we combine two discs with $\lambda_b\lambda_b$ and $\lambda_c\bar{\lambda}_c$ inserted, there will be no operators of mass dimension lower than or equal to 5. But if we take one disc with λ_b, λ_c and one with λ_b, $\bar{\lambda}_c$ we get the following operators:

$$\begin{array}{ll} Q^1_L U^{12}_R Q^1_L D_R & Q^1_L U^{12}_R L^{12} E^1_R \\ H_d Q^1_L U^3_R E^{23}_R & \end{array} \tag{6.1.11}$$

2. $\boxed{0,-2,1,0,-1}$

   If we combine $\lambda_b\lambda_b$ with $\bar{\lambda}_c\lambda_e$ there are no operators of dimension smaller or equal to 5. Combining $\lambda_b\bar{\lambda}_c$ and $\lambda_b\lambda_e$ we get the following operators:

$$\begin{array}{ll} Q^1_L D_R Q^1_L U^3_R & L^{12} E^1_R Q^1_L U^3_R \\ Q^1_L D_R H_u H_d & \end{array} \tag{6.1.12}$$

3. $\boxed{0,0,1,0,-1}$

   If we take the direct realization of the charges via $\bar{\lambda}_c\lambda_e$ we get the following operators:

$$\begin{array}{ll} D_R Q^1_L L^3 & H_u L^3 \\ E^1_R L^{12} Q^{23}_L U^3_R & E^{23}_R U^3_R Q^1_L L^3 \end{array} \tag{6.1.13}$$

   The first line would give dangerous R-parity violating couplings!

4. $\boxed{0, -2, 0, -1, 1}$

   Only the combination of discs with $\lambda_b \lambda_d$ and $\lambda_b \bar{\lambda}_e$ gives operators of dimension smaller or equal to 5. They are:

   $$L^{12} Q_L^1 U_R^{12} E_R^{23} \qquad\qquad L^{12} H_d E_R^{23} \tag{6.1.14}$$

   where the last one is also R-parity violating.

5. $\boxed{0, 0, 0, 1, -1}$

   In this case, if we take the direct realization of the charges via $\bar{\lambda}_d \lambda_e$ we get the following operators:

   $$E_R^1 U_R^{12} Q_L^1 L^{12} \qquad\qquad E_R^1 H_d Q_L^{23} U_R^3 \tag{6.1.15}$$

   For the second realization via $\bar{\lambda}_d \lambda_e \lambda_c \bar{\lambda}_c$ there are 3 different possibilities to distribute the zero modes. In the case of $\bar{\lambda}_d \lambda_e$ on the first and $\lambda_c \bar{\lambda}_c$ on the second disc there are no operators of dimension 5 or lower. The combination $\bar{\lambda}_d \lambda_c$ and $\lambda_e \bar{\lambda}_c$ also gives no interesting operators. Finally, the combination $\bar{\lambda}_d \bar{\lambda}_c$ and $\lambda_e \lambda_c$ gives the following operators:

   $$\begin{array}{ll} E_R^1 L^3 Q_L^1 U_R^{12} \qquad\qquad & E_R^1 L_3 H_d \\ E_R^1 U_R^3 Q_L^{23} H_d & \end{array} \tag{6.1.16}$$

Let us now focus on the possible contributions to flavour changing processes in the quark sector. Here we concentrate on neutral meson mixing because this is very well studied experimentally and they serve as one of the most promising examples of searches for signals of new physics.

Consider the contribution $Q_L^1 U_R^{12} Q_L^1 D_R$ from 6.1.11. Taking the F-term, it contains the following interaction (the tilde again denotes the superpartner to the fermion)

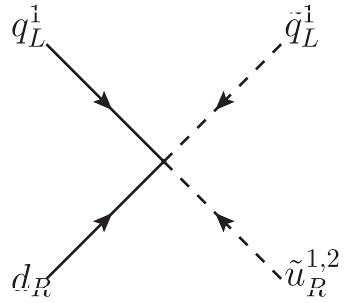


Figure 6.2: Instanton vertex: Solid lines represent fermions and dashed lines represent the bosonic superpartners

The strength of this vertex includes the instanton supression factor and for dimensional reasons the inverse of the characteristic mass scale, the string mass M_s combined with the right winding scale factor $\mathcal{V}^{\frac{1}{6}}$ (as was pointed out in chapter 4.2).

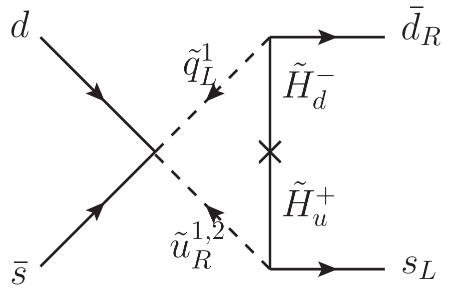


Figure 6.3: Coupling of the instanton contribution to the MSSM

How does this vertex couple to the MSSM to get a contribution to the neutral kaon mixing? One possibility is depicted in figure 6.3.

As a rough estimate of the strength of the amplitude of this process we just write down the characteristic contributions to the vertices and the propagators (since we are just interested in the order of magnitude of the process).

$$\mathcal{M} \approx \frac{e^{-S_0}}{M_s \mathcal{V}^{\frac{1}{6}}} \frac{m_{\text{susy}}^4 m_{K^0}^2 \mu \; m_d m_s}{m_{\tilde{q}_L^1}^2 m_{\tilde{u}_R^{12}}^2 m_{\tilde{H}}^2 m_{\text{weak}}^2} \tag{6.1.17}$$

Here we wrote a factor of m_x^2 for a propagator of a boson and m_x for the propagator of a fermion. We inserted a factor of the characteristic energy m_{susy}^4 coming from the loop integration. In addition there is a factor of the initial particle mass squared coming from the external legs and μ from the insertion of the Higgs μ-term.In addition we get two Yukawa couplings which are of the order $\frac{m_{\text{quark}}}{m_{\text{weak}}}$, where m_{weak} is the electroweak breaking scale. Finally M_s is the string mass scale which is combined with the winding scale factor $\mathcal{V}^{\frac{1}{6}}$ needed for the appropriate normalization for the superpotential.

However, if we want to have a really string theoretic contribution to neutral meson mixing, we should check if the whole process exists in string theory. Especially the loop should come from a one-loop diagram in open string theory. A possible realization of the whole process would be the combination of a disc and an annulus diagram as given in figure 6.4. As described in chapter 4.2, the integration over the zero modes leads to a connected diagram which results in the same topology as the MSSM process of figure 6.3 in the field theory limit.

But now we observe a problem. The out-states are fixed by the meson quark-content and therefore all matter branes on the outer boundary of the annulus. To get the right particles running in the loop, we have to fix the inner boundary to be the color brane a. But now we do not get the Higgs mass insertion any more. Instead we observe a state in the $a-a$-sector. It is not a gluon because it has to change flavour. One possibility would be exotic matter and therefore we can not estimate the strength of the amplitude. Taking the b-stack for the inner boundary of the annulus would give a weak gauge boson, a quark and a Higgs field in the loop. The result would be an effective instanton generated coupling of the form $\langle Q_L^1 D_R W H_d \rangle$ which is not gauge invariant and therefore also ruled out. The same argument holds for the c-stack as inner boundary. The remaining stacks would again give rise to exotic matter running in the loop.

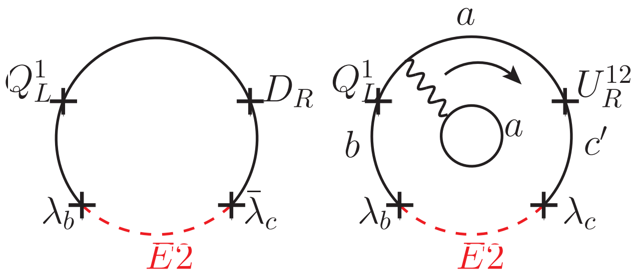


Figure 6.4: String theoretic realization of the process

However, there is also the possibility to realize the couplings in figure 6.3 nonperturbatively by D-instantons. In oder to do this one has to include the additional zero modes needed for these couplings in the annulus diagram in such a way that all charge selection rules are fulfilled. We leave this problem for future work, given in [46].

6.2 Lepton flavour violating

In the standard model with massless neutrinos, we have conserved lepton flavour number. Thus every observation of lepton-flavour violating (lfv) processes would be a definite hint for physics beyond the standard model. The most precise bounds on possible lfv processes come from the kaon and D-meson systems. In the following we list the most promising candidate decay processes (the branching ratio BR gives the ratio of the decay rate of the process to the total decay rate of the initial meson).

Table 6.2: Kaon decays into standard model particles with BR $< 5 \cdot 10^{-10}$, [44]

Decay	Branching ratio
$K_L^0 \to \mu^\pm e^\mp$	$< 4,7 \cdot 10^{-12}$
$K^+ \to \pi^+ \mu^+ e^-$	$< 2,8 \cdot 10^{-11}$
$K_L^0 \to \pi^0 \mu^+ e^-$	$< 4,4 \cdot 10^{-10}$
$K^+ \to \pi^- \mu^+ e^+$	$< 5,0 \cdot 10^{-10}$

An investigation of the above given summary of the systematic analysis of the instanton-induced phenomenology reveals the existence of various interesting operators which contain two quark and two lepton fields and therefore could contribute to one of the meson decays of the table. In the last section, we observed that there

Table 6.3: Examples for decay modes of D^0 with BR$< 5 \cdot 10^{-6}$ ([47])

Decay	Branching ratio
$D^0 \to e^+e^-$	$0,6 \cdot 10^{-6}$
$D^0 \to \mu^+\mu^-$	$3,4 \cdot 10^{-6}$
$D^0 \to \mu^\pm e^\mp$	$1,9 \cdot 10^{-6}$

is an additional limitation on possible processes if we want to realize them purely string theoretic, and therefore we will directly look for combinations of discs or discs with annuli. Taking the operator

$$Q^1_L U^{12}_R E^1_R L^{12} \tag{6.2.18}$$

we get the combination of a disc with an annulus which is given in figure 6.5. There are the following particles running in the loop (we use our example quiver of table 6.1): The bd'-sector corresponds to a lepton L, dd' is a gauge boson (photon) or its superpartner and the dc-sector corresponds to E^1_R. Therefore, in field theory the process would correspond to the Feynman diagram given in figure 6.6. To get proper MSSM couplings with standard model fermions as out-states, we get superpartners running in the loop: In our case sleptons $\tilde{L}$ and $\tilde{E}^1_R$ and the photino $\tilde{\gamma}$. The whole process would therefore contribute to the following decays of the D^0-meson:

$$D^0 \to e^- e^+ \quad D^0 \to \mu^- e^+ \tag{6.2.19}$$

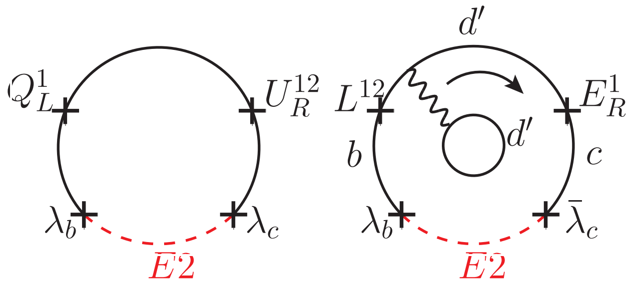


Figure 6.5: Instanton contribution to D^0-decay

Let us now estimate the order of magnitude of this process in field theory. From the instanton vertex we get the supression factor $\frac{e^{-S_0}}{M_s \mathcal{V}^{\frac{1}{6}}}$ which can be rewritten in terms of the Planck scale as $\frac{e^{-S_0}}{M_s^{\frac{2}{3}} \mathcal{M_P}^{\frac{1}{3}}}$. From the loop integration we get a factor of m^4_{susy}. Together with the propagator masses, the electromagnetic coupling constant and using as kinematical factor the mass of the initial state of the D^0 we get

$$\mathcal{M} \propto \frac{e^{-S_0}}{M_s^{\frac{2}{3}} M_P^{\frac{1}{3}}} m^4_{\text{susy}} \frac{m^2_{D^0}}{m^5_{\text{susy}}} g^2_{\text{em}}(m_{\text{susy}}) \tag{6.2.20}$$

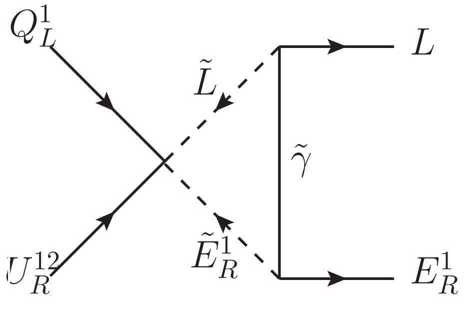


Figure 6.6: Instanton contribution to D^0-decay

Using 1TeV for the supersymmetry scale, $\frac{1}{100}$ for the instantonic supression (which is set by the ratio of the haviest quark to the charm quark) and the mass of the D^0 (2GeV), we get as a rough estimate

$$\mathcal{M} \propto 2 \cdot 10^{-14} \frac{TeV^{\frac{2}{3}}}{M_s^{\frac{2}{3}}} \tag{6.2.21}$$

We want to compare this to the experimental bound for the above given D^0-decays. The lifetime of the D^0 is $\tau_{D^0} = 4,1 \cdot 10^{-13} sec$ which gives us (using the branching ratio given in table 6.3) a decay rate of

$$\Gamma_{D^0 \to LE_R^1} = \mathrm{BR}(D^0 \to LE_R^1)\Gamma_{D^0} < 10^{-18}\mathrm{GeV}. \tag{6.2.22}$$

Now the amplitude of the process can be estimated by using

$$\Gamma_{D^0 \to LE_R^1} \approx m_{D^0} |\mathcal{M}|^2 \tag{6.2.23}$$

The result is $|\mathcal{M}| < 10^{-9}$. Comparing this with the field theory estimate of the instanton process we get a lower bound on the string scale of order 10^{-7} TeV which is not very restrictive.

We see that the winding factor gives us an additional very high supression factor such that instanton contributions to meson decay are very weak. To illustrate this, we skip this normalization factor and calculate only with the naive expectation of a supression of $\frac{1}{M_s}$ characteristic for effective dimension 5 operators, i.e. we have

$$\mathcal{M} \propto \frac{e^{-S_0}}{M_s} m_{\mathrm{susy}}^4 \frac{m_{D^0}^2}{m_{\mathrm{susy}}^5} g_{\mathrm{em}}^2(m_{\mathrm{susy}}) = 4 \cdot 10^{-9} \frac{TeV}{M_s} \tag{6.2.24}$$

which would lead to the lower bound of $M_s > 2$ TeV. Another important fact is the experimental bound on the D^0-decay, which is not as good as for kaon decays. If we had experimental bounds similar to kaon decays (table 6.2), we would get $M_s > 3 \cdot 10^{-3}$ TeV with winding scale and $M_s > 2 \cdot 10^3$ TeV without this scale. We conclude that instanton generated contributions to flavour violating processes give weaker bounds on the string scale in our example model as for example the perturbatively realized coupling $\langle \bar{D}_R D_R \bar{D}_R D_R \rangle$.

Chapter 7

Summary and outlook

At the beginning of this work, a brief introduction to the basics of Calabi-Yau and orientifold compactification and a discussion of the most important properties and consistency conditions of intersecting D-brane models was given. Tadpole/anomaly cancellation resulted in important constraints on the geometry of intersecting branes (and therefore on the particle content of the brane realization of the standard model or the MSSM). We then introduced D-brane instantons by first studying the D(-1)/D(3)-system. Here one can observe that D(-1) branes filling the external Minkowski space can be interpreted as gauge instantons, especially by noting that we could get the ADHM constraints and the classical instanton solution by considering scattering of open strings ending on the D(-1)/D(3)-branes.

Generalizing this idea, we were traced to the consideration of Euclidean branes in the internal Calabi-Yau manifold, which are pointlike in the external space. Using an analogy to the field theory prescription of integrating over the space of collective coordinates, we were able to compute instanton corrections to the effective four-dimensional superpotential of the compactified type IIA theory.

We realized that these corrections can be used to generate perturbatively forbidden but phenomenologically desired Yukawa couplings in MSSM orientifold models. Due to the instanton-supression factor, depending on the volume of the internal cycle, it was possible to get realistic mass hierarchies for the different families of particles. But we also observed that these “Yukawa-instantons” not only generate the Yukawa couplings but also give rise to other superpotential couplings, like baryon-number-violating terms leading for example to proton decay. In order not to violate experimental bounds and still to get realistic mass hierarchies, we presented 5-stack models of the MSSM (first found in [41]) whose Yukawa couplings do not generate such catastrophic effects.

But even in the absence of proton decay operators there is still the possibility of flavour violating effects in the quark and lepton sector. On the one hand, there are perturbatively allowed flavour violating effects firstly discussed in [49]. In addition we found in the last chapter (after performing a systematic analysis of instanton-generated operators of mass dimension smaller or equal to 5, presented in the appendix) the presence of lepton flavour violating effects generated by D-instantons. Especially we found a contribution of a disc and an annulus diagram to the decay of the neutral D-meson. Due to the experimentally measured supression of this process, it was possible to get a lower bound on the string scale. This lower bound is weaker than the bound which one gets by considering perturbatively allowed contributions to neutral kaon mixing, mainly because of the winding-scale supression of superpotential couplings and the weak experimental bound which will be definitely improved at the Large Hadron Collider in the future.

It would be interesting to find an MSSM orientifold model which also does not allow the above mentioned perturbatively allowed flavour violating operators. These models would be interesting candidates for models with a low string scale (and therefore very exciting for future experiments at the LHC) and the instanton contributions to flavour violating would give very important lower bounds on the string scale in such models. We leave it for future research to check if such models are possible (passing for example the tadpole/anomaly constraints). In this case it is important to determine the lower bounds on the string scale coming from instanton contributions to flavour violating effects as discussed in the last chapter.

Appendix A

Mathematical definitions

A.1 $\hat{\mathcal{A}}$-class, Hirzebruch $\mathcal{L}$-class and the Chern character

In the first appendix we want to give the definitions of important geometrical quantities used in the main text. To write down the Chern-Simons couplings of D-branes and O-planes, we used the Chern character, the $\hat{\mathcal{A}}$-class and the Hirzebruch -$\mathcal{L}$ -class. We now want to state briefly the geometric background.

A.1.1 Multiplicative sequences

Let $E \longrightarrow X$ be a complex vector bundle of rank n over a manifold X. Let us denote by $c(E)$ the total Chern class

$$c(E) = 1 + c_1(E) + c_2(E) + \cdots + c_n(E) \tag{A.1.1}$$

The Chern classes c_k are defined as follows. Let $P \xrightarrow{\pi} X$ be the associated $Gl(n, \mathbb{C})$ principal bundle to E and Ω its curvature. Then we can express the following determinant in terms of the pullbacks of forms on X

$$\det(\mathbf{1}_n - \frac{1}{2\pi i}\Omega) = \pi^*(1 + \alpha_1 + \cdots + \alpha_n) \tag{A.1.2}$$

and $c_l(E)$ is the cohomology class of the form α_l.

One can show that the total Chern class has the following useful property:

$$c(E \oplus E') = c(E)c(E') \tag{A.1.3}$$

As an application of this, let us assume we can split the bundle E into a sum of complex line bundles $E \cong L_1 \oplus \cdots \oplus L_n$, then we get from A.1.3

$$c(E) = \prod_{k=1}^{n}(1 + x_k) \quad x_k = c_1(L_k) \tag{A.1.4}$$

Let us also define the rational Pontrjagin classes. For this E should be a real oriented vector bundle of dimension $2n$ and we define the rational Pontrjagin classes by

$$p_j(E) = (-1)^j c_{2j}(E \otimes \mathbb{C}) \tag{A.1.5}$$

and the total Pontrjagin class is given by

$$p(E) = 1 + p_1(E) + p_2(E) + \cdots + p_n(E) \tag{A.1.6}$$

It has the same property A.1.3 as the total Chern class. As a similar application we assume now that the complexification $E \otimes \mathbb{C}$ can be decomposed as a sum of line bundles

$$E \otimes \mathbb{C} \cong L_1 \oplus \bar{L}_1 \oplus \cdots \oplus L_n \oplus \bar{L}_n \tag{A.1.7}$$

and we get for the total Pontrjagin class

$$p(E) = \prod_{k=1}^{n} (1 + x_k^2) \tag{A.1.8}$$

As a next step we introduce the notion of a multiplicative sequence. Let $\hat{\mathbb{Q}}[[x]]$ denote the set of formal power series with variable x and constant term 1. For an element $g(x) \in \hat{\mathbb{Q}}[[x]]$ let us consider the expression in k variables $g(x_1) \cdots g(x_k)$ which is symmetric in the x_l and therefore can be expanded in terms of the elementary symmetric functions $\sigma_r(x_1, \ldots, x_k) = \sum_{i_1 < \cdots < i_r} x_{i_1} \cdots x_{i_r}$:

$$g(x_1) \cdots g(x_k) = 1 + F_1(\sigma_1) + F_2(\sigma_1, \sigma_2) + F_3(\sigma_1, \sigma_2, \sigma_3) + \ldots \tag{A.1.9}$$

Where the $F_i(\sigma_1, \ldots, \sigma_i)$ are polynomials (one can show that they are well defined independent of the number of variables $x_1, \ldots, x_k$) and define the so-called multiplicative sequence $\{F_l(\sigma_1, \ldots, \sigma_l)\}_{l=1}^{\infty}$ determined by the power series $g(x)$.

If we now have a commutative algebra A with unit over $\mathbb{Q}$ which allows a decomposition $A = A^0 \oplus A^1 \oplus A^2 \oplus \ldots$ and $A^m A^n \subseteq A^{m+n}$, let us define the set $\hat{A}$ of formal sums

$$1 + a_1 + a_2 + \ldots \quad a_i \in A^i \tag{A.1.10}$$

with the following multiplication rule

$$(1 + a_1 + a_2 + \ldots)(1 + b_1 + b_2 + \ldots) = 1 + (a_1 + b_1) + (a_2 + a_1 b_1 + b_2) + \ldots \tag{A.1.11}$$

i.e. we collect the terms of equal grading. With this multiplication $\hat{A}$ turns into an Abelian group. If we now have a multiplicative sequence as given above, we get a group homomorphism

$$\begin{aligned} \mathcal{F} :& \hat{A} \to \hat{A} \\ & 1 + a_1 + a_2 + \ldots \mapsto 1 + F_1(a_1) + F_2(a_1, a_2) + \ldots \end{aligned} \tag{A.1.12}$$

which gives for $\hat{A} = \hat{\mathbb{Q}}$ and the multiplicative sequence determined by $g(x)$:

$$\mathcal{F}(1+\sigma_1+\sigma_2+\cdots+\sigma_n) = g(x_1)g(x_2)\cdots g(x_k) \tag{A.1.13}$$

as we can see from the expansion A.1.9.

If we take as algebra the cohomology groups $H^{2^*}(X,\mathbf{Q})$, we can define the total $\mathcal{F}$-class of the bundle E as $\mathcal{F}(c(E))$. From the property A.1.3 and from the homomorphism-property of $\mathcal{F}$ we get (if we use the Splitting principle [16])

$$\begin{aligned}\mathcal{F}(c(E)) =&\mathcal{F}(\prod_{i=1}^{n}(1+c_1(L_i)))\\ =&\mathcal{F}(1+\sigma_1(c_1(L_1),\ldots,c_1(L_n))+\sigma_n(c_1(L_1),\ldots,c_n(L_n)))\\ =&g(c_1(L_1))\cdots g(c_1(L_N))\end{aligned} \tag{A.1.14}$$

The same statements hold for real bundles and multiplicative sequences of Pontrjagin classes.

We are now finally able to write down the classes we used in the main text. If we take as formal power series

$$\hat{a}(x) = \frac{\sqrt{x}/2}{\sinh(\sqrt{x}/2)} = 1 - \frac{1}{24}x + \frac{7}{2^7\cdot 3^2\cdot 5}x^2 + \ldots \tag{A.1.15}$$

we get as multiplicative sequence the $\hat{\mathcal{A}}$-sequence. Writing $x_k = c_1(L_k)$ we get for the total $\hat{\mathcal{A}}$-class

$$\begin{aligned}\hat{\mathcal{A}}(E) =&\prod_j \frac{x_j/2}{\sinh(x_j/2)}\\ =&1+\hat{\mathcal{A}}_1(\sigma_1(x_1^2,\ldots,x_n^2))+\hat{\mathcal{A}}_2(\sigma_1(x_1^2,\ldots,x_n^2),\sigma_2(x_1^2,\ldots,x_n^2))+\ldots\end{aligned} \tag{A.1.16}$$

where the the first three terms in the multiplicative sequence are given by

$$\begin{aligned}\hat{\mathcal{A}}_1(p_1) &= -\frac{1}{24}p_1\\ \hat{\mathcal{A}}_2(p_1,p_2) &= \frac{1}{2^7\cdot 2^3\cdot 5}(-4p_2+7p_1^2)\\ \hat{\mathcal{A}}_3(p_1,p_2,p_3) &= -\frac{1}{2^{10}\cdot 3^3\cdot 5\cdot 7}(16p_3-44p_2p_1+31p_1^3)\end{aligned} \tag{A.1.17}$$

Taking for the formal power series the following

$$l(x) = \frac{\sqrt{x}}{\tanh(\sqrt{x})} = 1+\frac{1}{3}x-\frac{1}{45}x^2+\ldots \tag{A.1.18}$$

we get the Hirzebruch-$\mathfrak{L}$-sequence

$$\mathfrak{L}(E) = \prod_j \frac{x_j}{\tanh(x_j)} \tag{A.1.19}$$

A.1.2 The Chern character

We finally want to give the definition and the easiest properties of the Chern character. If we write the total Chern class of a complex vector bundle E formally as a product

$$c(E) = 1 + c_1(E) + c_2(E) + \cdots + c_n(E) = \prod_{k=1}^{n}(1 + x_k) \tag{A.1.20}$$

we can define the Chern character by [16]

$$\mathrm{ch}(E) := e^{x_1} + \cdots + e^{x_n} \tag{A.1.21}$$

It has the following behaviour for direct sums and tensor products of vector bundles:

$$\begin{aligned} \mathrm{ch}(E \oplus E') &= \mathrm{ch}(E) + \mathrm{ch}(E') \\ \mathrm{ch}(E \otimes E') &= \mathrm{ch}(E)\mathrm{ch}(E') \end{aligned} \tag{A.1.22}$$

Appendix B

Clifford algebra, t'Hooft tensors and spin field correlators

In this appendix the main conventions used in chapter 4 will be explained briefly and the calculational tools will be stated. We begin with the conventions for the Euclidean $d = 4$ Clifford algebra and the t'Hooft symbols. A representation of the Euclidean Lorentz group $SO(4)$ is given in terms of the identity and the Pauli matrices

$$\sigma^\mu = (\mathbf{1}, \vec{\tau}) \qquad \bar{\sigma}_\mu = \sigma_\mu^\dagger = (\mathbf{1}, i\vec{\tau}) \tag{B.0.1}$$

which fulfil the Clifford algebra

$$\sigma_\mu \bar{\sigma}_\nu - \sigma_\nu \bar{\sigma}_\mu = 2\delta_{\mu\nu}\mathbf{1} \tag{B.0.2}$$

and act on two component spinors $\psi = \binom{\psi_\alpha}{\psi^{\dot{\alpha}}}$. The Lorentz generators are given by

$$\sigma_{\mu\nu} = \frac{1}{2}(\sigma_\mu \bar{\sigma}_\nu - \sigma_\nu \bar{\sigma}_\mu) \qquad \bar{\sigma}_{\mu\nu} = \frac{1}{2}(\bar{\sigma}_\mu \sigma_\nu - \bar{\sigma}_\nu \sigma_\mu) \tag{B.0.3}$$

In ten dimensions the spinor representation of the Lorentz group has dimension 32 and the corresponding gamma-matrices can be decomposed into dimension four and six gamma matrices in the following way :

$$\begin{aligned} \Gamma^\mu_{32} &= \gamma_4 \otimes \mathbf{1}_8 \qquad \mu \in \{1, \dots, 4\} \\ \Gamma^a_{32} &= \gamma_4^5 \otimes \Gamma^a_8 \qquad a \in \{5, \dots, 10\} \end{aligned} \tag{B.0.4}$$

where the subscripts denote the dimensionality of the representation, which is $2^{d/2}$ if spacetime is d-dimensional.

The dimension six gamma matrices Γ^a_8 are given by

$$\Gamma^a_8 = \begin{pmatrix} 0 & \Sigma^a \\ \bar{\Sigma}^a & 0 \end{pmatrix} \tag{B.0.5}$$

where the Σ-matrices obey the six-dimensional Clifford algebra. They act on $SO(6)$-spinors $\binom{\Lambda^A}{\Lambda_A}$.

The t'Hooft symbols are defined by

$$\begin{aligned}(\sigma_{\mu\nu})_\alpha^{\ \beta} &= i\eta^c_{\mu\nu}(\tau^c)_\alpha^{\ \beta}\\ (\bar{\sigma}_{\mu\nu})^{\dot{\alpha}}_{\ \dot{\beta}} &= i\bar{\eta}^c_{\mu\nu}(\tau^c)^{\dot{\alpha}}_{\ \dot{\beta}}\end{aligned} \tag{B.0.6}$$

they can be explicitly written down in terms of antisymmetric and symmetric symbols:

$$\begin{aligned}&\eta^c_{\mu\nu} = \bar{\eta}^c_{\mu\nu} = \epsilon_{c\mu\nu}, \quad \mu,\nu \in \{1,2,3\}\\ &\bar{\eta}^c_{4\nu} = -\eta^c_{4\nu} = \delta^c_\nu\\ &\eta^c_{\mu\nu} = -\eta^c_{\nu\mu}, \quad \bar{\eta}^c_{\mu\nu} = -\bar{\eta}^c_{\mu\nu}\end{aligned} \tag{B.0.7}$$

Now let us finally write down the technical tools given by correlation functions which were used in chapter 4 to calculate the tree level amplitudes. Let us denote the fermionic part of the superstring by ψ_μ (which is needed also for vertex operators in the Neveu-Schwarz sector) and the spin field (needed for vertex operators in the Ramond sector by S^α, S^A (we already performed the splitting into four and six dimensional fields), we used the following correlators (a systematic survey of these results can be found in [48]):

$$\begin{aligned}\langle S^{\dot{\alpha}}(z_1)\psi_\mu(z_2)S_\beta(z_3)\rangle =& \frac{1}{\sqrt{2}}(\bar{\sigma}_\mu)^{\dot{\alpha}}{}_\beta(z_1-z_2)^{-\frac{1}{2}}(z_2-z_3)^{-\frac{1}{2}}\\ \langle S^{\dot{\alpha}}(z_1)\psi_\mu\psi_\nu(z_2)S^{\dot{\beta}}(z_3)\rangle =& -\frac{1}{2}(\bar{\sigma}_{\mu\nu})^{\dot{\alpha}\dot{\beta}}(z_1-z_3)^{\frac{1}{2}}(z_1-z_2)^{-1}(z_2-z_3)^{-1}\\ \langle S^A(z_1)\psi^a(z_2)S^B(z_3)\rangle =& \frac{i}{\sqrt{2}}(\Sigma^a)^{AB}(z_1-z_2)^{-\frac{1}{2}}(z_1-z_3)^{-\frac{1}{4}}(z_2-z_3)^{-\frac{1}{2}}\\ \langle S_A(z_1)\psi^a(z_2)S_B(z_3)\rangle =& -\frac{i}{\sqrt{2}}(\Sigma^a)_{AB}(z_1-z_2)^{-\frac{1}{2}}(z_1-z_3)^{-\frac{1}{4}}(z_2-z_3)^{-\frac{1}{2}}\end{aligned} \tag{B.0.8}$$

In addition to that, we used boundary changing operators for the open strings in the $D(-1)-D3$ system because of the mixed Dirichlet-Neumann boundary conditions. Their correlators can be calculated by using the operator product expansions:

$$\Delta(z_1)\bar{\Delta}(z_2) \cong (z_1-z_2)^{\frac{1}{2}} \qquad \bar{\Delta}(z_1)\Delta(z_2) \cong -(z_1-z_2)^{\frac{1}{2}} \tag{B.0.9}$$

where the minus sign is the rule to get the right spacetime statistics in the computation of correlators.

Appendix C

Systematic analysis of the 5-stack model

In the following tables we will systematically write down all holomorphic chain products of matter superfields that are allowed by the global $U(1)$-symmetries if we combine them with the zero modes given in the headline of the tables. The zero modes come from the intersection of an instantonic E2-brane with the matter stacks of the 5-stack quiver given in table 6.1. These instantons are needed to generate non-perturbatively the quark and lepton Yukawa couplings which are also listed. In some exceptional cases we also listed non-holomorphic products (labelled by "anti").

Quark Yukawa couplings

$$\boxed{Q_L^1 U_R^{12} H_u}$$

charge: $0\ -2\ 0\ 0\ 0$

zero modes : $\lambda_b\ \lambda_b$ and $\lambda_b\ \lambda_b\ \lambda_c\ \bar{\lambda}_c$

Table C.1: $\lambda_b\ \lambda_b$

b	$\bar{b}a$	$\bar{a}c$	$\bar{c}\bar{a}$	$a\bar{b}$	$Q_L^1 D_R U_R^{12} Q_L^1$
			$\bar{c}\bar{b}$	x	$Q_L^1 D_R H_d$
		$\bar{a}\bar{c}$	$c\bar{a}$	$a\bar{b}$	$Q_L^1 U_R^{12} D_R Q_L^1$
			$c\bar{b}$	x	$Q_L^1 U_R^{12} H_u$
			cd	$\overline{db}$	$Q_L^1 U_R^{12} E_R^1 L^{12}$
			ce	$\bar{e}\bar{b}$	No
		$\bar{a}\bar{e}$	ec	$\bar{c}\bar{b}$	$Q_L^1 U_R^3 E_R^{23} H_d$
	$\bar{b}c$	$\bar{c}\bar{a}$	$a\bar{b}$	x	$H_u U_R^{12} q_L^1$
			ab		No
		$\bar{c}\bar{d}$	ba	$\bar{a}\bar{b}$	anti
			$b\bar{e}$	$e\bar{b}$	anti
		$\bar{c}\bar{b}$	x	x	$H_u H_d$
	$\bar{b}\bar{c}$	$c\bar{a}$	$a\bar{b}$	x	$H_d D_R Q_L^1$
			ab		No
		$c\bar{b}$	b	$\bar{a}\bar{b}$	anti
		$c\bar{b}$	x	x	$H_u H_d$
		cd	$\overline{db}$	x	$H_d E_R^1 L^{12}$
		ce	$\bar{e}\bar{a}$	$a\bar{b}$	$H_d E_R^{23} U_R^3 Q_L^1$
			$\bar{e}b$	x	No
	$\bar{b}\bar{d}$	dc	$\bar{c}\bar{a}$	$a\bar{b}$	$L^{12} E_R^1 U_R^{12} Q_L^1$
			$\bar{c}\bar{b}$	x	$L^{12} E_R^1 H_d$

$\boldsymbol{\lambda_b\ \lambda_b\ \lambda_c\bar{\lambda}_c}$

Table C.2: $\lambda_b\ \lambda_c$

b	$\bar{b}a$	$\bar{a}c$	$\bar{c}c$	No
		$\bar{a}\bar{c}$	x	$Q_L^1 U_R^{12}$
		$\bar{a}\bar{e}$	$e\bar{c}$	No
	$\bar{b}c$	$\bar{c}\bar{a}$	$a\bar{c}$	anti
		$\bar{c}\bar{b}$	$b\bar{c}$	anti
	$\bar{b}\bar{c}$	$c\bar{a}$	$a\bar{c}$	anti
		$c\bar{b}$	$b\bar{c}$	anti
		cd	$\bar{d}\bar{c}$	No
		ce	$\bar{e}\bar{c}$	No
		x	x	H_d
	$\bar{b}\bar{d}$	dc	x	No

Table C.3: $\lambda_b\bar{\lambda}_c$

b	$\bar{b}a$	$\bar{a}c$	x	$Q_L^1 D_R$
		$\bar{a}\bar{c}$	x	No
		$\bar{a}\bar{e}$	ec	$Q_L^1 U_R^3 E_R^{23}$
	$\bar{b}c$	x	x	H_u
		$\bar{c}\bar{a}$	ac	anti
		$\bar{c}\bar{b}$	bc	anti
	$\bar{b}\bar{c}$	$c\bar{a}$	ac	anti
		$c\bar{b}$	bc	anti
		cd	$\bar{d}c$	No
		ce	$\bar{e}c$	No
	$\bar{b}\bar{d}$	dc	x	$L^{12} E_R^1$

$$\boxed{Q_L^1 U_R^3 H_u}$$

charge: $0\ -2\ 1\ 0\ -1$

zero modes: $\lambda_b\lambda_b\bar{\lambda}_c\lambda_e$

Table C.4: $\bar{\boldsymbol{\lambda}}_{\mathbf{c}}\boldsymbol{\lambda}_{\mathbf{e}}$

$\bar{c}$	$c\bar{a}$	$a\bar{b}$	ba	$a\bar{e}$	No
			$b\bar{e}$	x	$D_R Q_L^1 L^3$
		ab	$\bar{b}a$	$a\bar{e}$	No
			$\bar{b}c$	$\bar{c}\bar{e}$	anti
			$\bar{b}\bar{c}$	$c\bar{e}$	No
			$\bar{b}\bar{d}$	$d\bar{e}$	No
	$c\bar{b}$	ba	$\bar{a}c$	$\bar{c}\bar{e}$	anti
			$\bar{a}\bar{c}$	$c\bar{e}$	No
			$\bar{a}\bar{e}$	x	$H_u Q_L^{23} U_R^3$
		$b\bar{e}$	ec	$\bar{c}\bar{e}$	anti
		$b\bar{e}$	x	x	$H_u L^3$
	cd	$\overline{db}$	ba	$\bar{a}\bar{e}$	$E_R^1 L^{12} Q_L^{23} U_R^3$
			$b\bar{e}$	x	$E_R^1 L^{12} L^3$
	ce	$\bar{e}b$	$\bar{b}a$	$a\bar{e}$	No
			$\bar{b}c$	$\bar{c}\bar{e}$	anti
			$\bar{b}\bar{c}$	$c\bar{e}$	No
			$\bar{b}\bar{d}$	$d\bar{e}$	No
		$\bar{e}\bar{a}$	ab	$\bar{b}\bar{e}$	No
			$a\bar{b}$	$b\bar{e}$	$E_R^{23} U_R^3 Q_L^1 L^3$

$$\boxed{Q_L^1 D_R H_d}$$

Table C.5: $\lambda_{\mathbf{b}}\lambda_{\mathbf{e}}$

b	$\bar{b}a$	$\bar{a}c$	$\bar{c}\bar{a}$	$a\bar{e}$	No
			$\bar{c}\bar{b}$	$b\bar{e}$	$Q_L^1 D_R H_d L^3$
		$\bar{a}\bar{c}$	$c\bar{a}$	$a\bar{e}$	No
			$c\bar{b}$	$b\bar{e}$	$Q_L^1 U_R^{12} H_u L^3$
			cd	$\bar{d}\bar{e}$	No
			ce	x	No
		$\bar{a}\bar{e}$	x	x	$Q_L^1 U_R^3$
	$\bar{b}c$	$\bar{c}\bar{a}$	$\bar{a}\bar{e}$	x	$H_u U_R^{12} U_R^3$
		$\bar{c}\bar{b}$	$b\bar{e}$	x	$H_u H_d$
	$\bar{b}\bar{c}$	$c\bar{a}$	$a\bar{e}$		No
		$c\bar{b}$	$b\bar{e}$	x	$H_d H_u L^3$
		cd	$\bar{d}\bar{e}$		No
		ce	x		No
	$\bar{b}\bar{d}$	dc	$\bar{c}\bar{e}$		No

charge: $0 \ -2\ 0\ 0\ 0$

phenomenology already discussed

$$\boxed{Q_L^{23} D_R H_d}$$

charge: $0\ 0\ 0\ 0\ 0$

perturbatively allowed.

$$\boxed{Q_L^{23} U_R^{12} H_u}$$

charge: $0\ 0\ 0\ 0\ 0$

therefore perturbatively allowed.

$$\boxed{Q_L^{23} U_R^{3} H_u}$$

charge: $0\ 0\ 1\ 0\ -1$

zero modes: $\bar{\lambda}_c \lambda_e$

this sector was already checked.

other realization: $\lambda_b \bar{\lambda}_b \bar{\lambda}_c \lambda_e$ if possible.

Table C.6: $\bar{\lambda}_{\mathbf{b}}\lambda_{\mathbf{e}}$

$\bar{b}$	ba	$\bar{a}c$	$\bar{c}\bar{a}$	$a\bar{e}$	No
			$\bar{c}\bar{b}$	$b\bar{e}$	$Q_L^{23}D_RH_dL^3$
		$\bar{a}\bar{c}$	$c\bar{a}$	$a\bar{e}$	No
			$c\bar{b}$	$b\bar{e}$	$Q_L^{23}U_R^{12}H_uL^3$
			cd	$\bar{d}\bar{e}$	No
			ce	x	No
		$\bar{a}\bar{e}$	x	x	$Q_L^{23}U_R^3$
	$b\bar{e}$	x	x	x	L^3
		ec	$\bar{c}\bar{a}$	$a\bar{e}$	No
			$\bar{c}\bar{b}$	$b\bar{e}$	$L^3E_R^{23}H_dL^3$

Lepton Yukawa couplings

$\boxed{L^{12}H_dE_R^1}$

charge: $0\ -2\ 0\ 0\ 0$

phenomenology already discussed.

$\boxed{L^{12}H_dE_R^{23}}$

charge: $0\ -2\ 0\ -1\ 1$

zero modes: $\lambda_b, \lambda_b, \lambda_d, \bar{\lambda}_e$

Table C.7: $\lambda_{\mathbf{d}}\lambda_{\mathbf{e}}$

d	$\overline{db}$	ba	$\bar{a}\bar{e}$	$L^{12}Q_L^{23}U_R^3$
		$b\bar{e}$	x	$L^{12}L^3$

$\boxed{L^3H_dE_R^{23}}$

charge: $0\ 0\ 0\ 0\ 0$

therefore perturbatively allowed.

$\boxed{L^3H_dE_R^1}$

charge: $0\ 0\ 0\ 1\ -1$

zero modes: $\bar{\lambda}_d, \lambda_e$

second realization: $\bar{\lambda}_d, \lambda_e, \lambda_c, \bar{\lambda}_c$

Table C.8: $\lambda_{\mathbf{b}}, \lambda_{\mathbf{d}}$

b	$\bar{b}a$	$\bar{a}c$	$\bar{c}\bar{d}$	No
		$\bar{a}\bar{c}$	$c\bar{d}$	No
		$\bar{a}\bar{e}$	$e\bar{d}$	No
	$\bar{b}c$	$\bar{c}\bar{a}$	$a\bar{d}$	No
		$\bar{c}\bar{b}$	$b\bar{d}$	No
	$\bar{b}\bar{c}$	$c\bar{a}$	$a\bar{d}$	No
		$c\bar{b}$	$b\bar{d}$	No
		cd	x	No
		ce	$\bar{e}\bar{d}$	No
	$\bar{b}\bar{d}$	x	x	L^{12}
		dc	$\bar{c}\bar{d}$	No

Table C.9: $\lambda_{\mathbf{b}}\bar{\lambda}_{\mathbf{e}}$

b	$\bar{b}a$	$\bar{a}c$	$\bar{c}\bar{a}$	ae	No
			$\bar{c}\bar{b}$	be	No
		$\bar{a}\bar{c}$	$c\bar{a}$	ae	No
			$c\bar{b}$	be	No
			cd	$\bar{a}e$	No
			ce	x	$Q_L^1 U_R^{12} E_R^{23}$
		$\bar{a}\bar{e}$	ec	$\bar{c}e$	No
	$\bar{b}c$	$\bar{c}\bar{a}$	$a\bar{b}$	be	No
			ab	$\bar{b}e$	No
		$\bar{c}\bar{b}$	ba	$\bar{a}e$	No
			$b\bar{e}$	x	No
	$\bar{b}\bar{c}$	$c\bar{a}$	$a\bar{b}$	be	No
		$c\bar{b}$	ba	$\bar{a}e$	No
			$b\bar{e}$	x	No
		cd	$\overline{db}$	be	No
		ce	x	x	$H_d E_R^{23}$
	$\bar{b}\bar{d}$	dc	$\bar{c}\bar{a}$	ae	No
			$\bar{c}\bar{b}$	be	No

Table C.10: $\bar{\lambda}_{\mathbf{d}}, \lambda_{\mathbf{e}}$

$\bar{d}$	dc	$\bar{c}\bar{a}$	$a\bar{b}$	$b\bar{e}$	$E_R^1 U_R^{12} Q_L^1 L^{12}$
			ab	$\bar{b}\bar{e}$	No
		$\bar{c}\bar{b}$	ba	$\bar{a}\bar{e}$	$E_R^1 H_d Q_L^{23} U_R^3$
			$b\bar{e}$	x	$E_R^1 H_d L^3$

Table C.11: $\bar{\lambda}_d, \lambda_c$

$\bar{d}$	dc	$\bar{c}\bar{a}$	$a\bar{c}$	x	No
			$a\bar{b}$	$\bar{b}\bar{c}$	$E_R^{23}U_R^{12}Q_L^1H_d$
			ab	$b\bar{c}$	No
		$\bar{c}\bar{b}$	ba	$a\bar{c}$	anti
			$b\bar{e}$	$e\bar{c}$	No

Table C.12: $\bar{\lambda}_d\bar{\lambda}_c$

$\bar{d}$	dc	x	x	x	E_R^1
		$\bar{c}\bar{a}$	$a\bar{b}$	$\bar{b}c$	$E_R^1U_R^{12}Q_L^1H_u$
			ab	bc	No

Table C.13: $\lambda_e\lambda_c$

e	$\bar{e}b$	$\bar{b}a$	$\bar{a}c$	x	No
			$\bar{a}\bar{c}$	x	$L^3Q_L^1U_R^{12}$
			$\bar{a}\bar{e}$	$e\bar{c}$	No
		$\bar{b}c$	$\bar{c}\bar{a}$	$a\bar{c}$	anti
			$\bar{c}\bar{b}$	$b\bar{c}$	anti
		$\bar{b}\bar{c}$	x	x	L^3H_d
			$c\bar{a}$	$a\bar{c}$	anti
			$c\bar{b}$	$b\bar{c}$	anti
		$\bar{b}\bar{d}$	cd	$\bar{d}\bar{c}$	No
			ce	$\bar{e}\bar{c}$	No
			dc	x	No
	$\bar{e}\bar{a}$	$a\bar{b}$	ba	$\bar{a}\bar{c}$	$U_R^3Q_L^1Q_L^{23}U_R^{12}$
			$b\bar{e}$	$e\bar{c}$	No
		ab	$\bar{b}a$	$\bar{a}\bar{c}$	$U_R^3Q_L^{23}Q_L^1U_R^{12}$
			$\bar{b}\bar{c}$	x	$U_R^3Q_L^{23}H_d$
			$\bar{b}\bar{d}$	$d\bar{c}$	No
			$\bar{b}c$	x	No

Table C.14: $\lambda_c\bar{\lambda}_c$

c	$\bar{c}\bar{a}$	$a\bar{b}$	ba	$\bar{a}c$	$U_R^{12}Q_L^1Q_L^{23}D_R$
			$b\bar{e}$	ec	$U_R^{12}Q_L^1L^3E_R^{23}$
		ab	$\bar{b}a$	$\bar{a}c$	$U_R^{12}Q_L^{23}Q_L^1D_R$
			$\bar{b}c$	x	$U_R^{12}Q_L^{23}H_u$
			$\bar{b}\bar{c}$	x	No
			$\bar{b}\bar{d}$	dc	$U_R^{12}Q_L^{23}L^{12}E_R^1$
	$\bar{c}\bar{b}$	ba	$\bar{a}c$	x	$H_dQ_L^{23}D_R$
			$\bar{a}\bar{c}$	x	No
			$\bar{a}\bar{e}$	ec	$H_dQ_L^{23}U_R^3E_R^{23}$
		$b\bar{e}$	ec	x	$H_dL^3E_R^{23}$

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